Generating correct code for your programmers

PLISS 2025 – Part I

Hila Peleg - Technion

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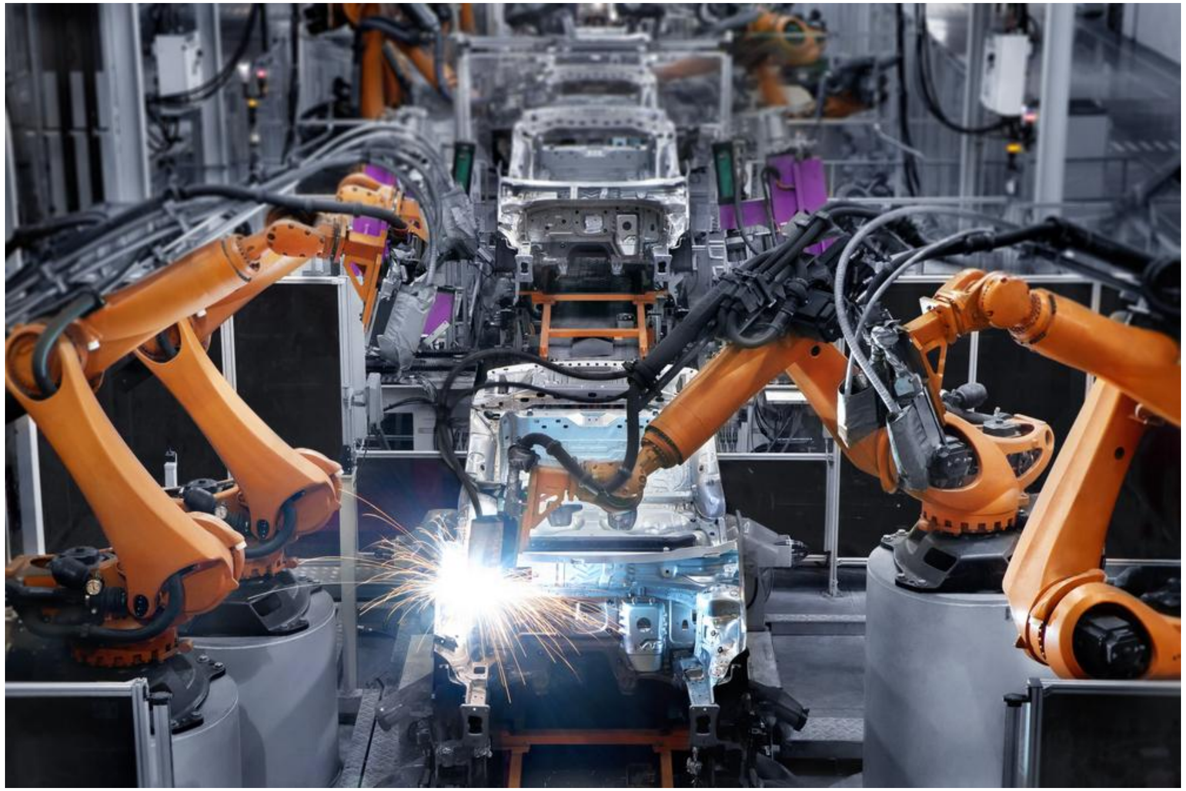
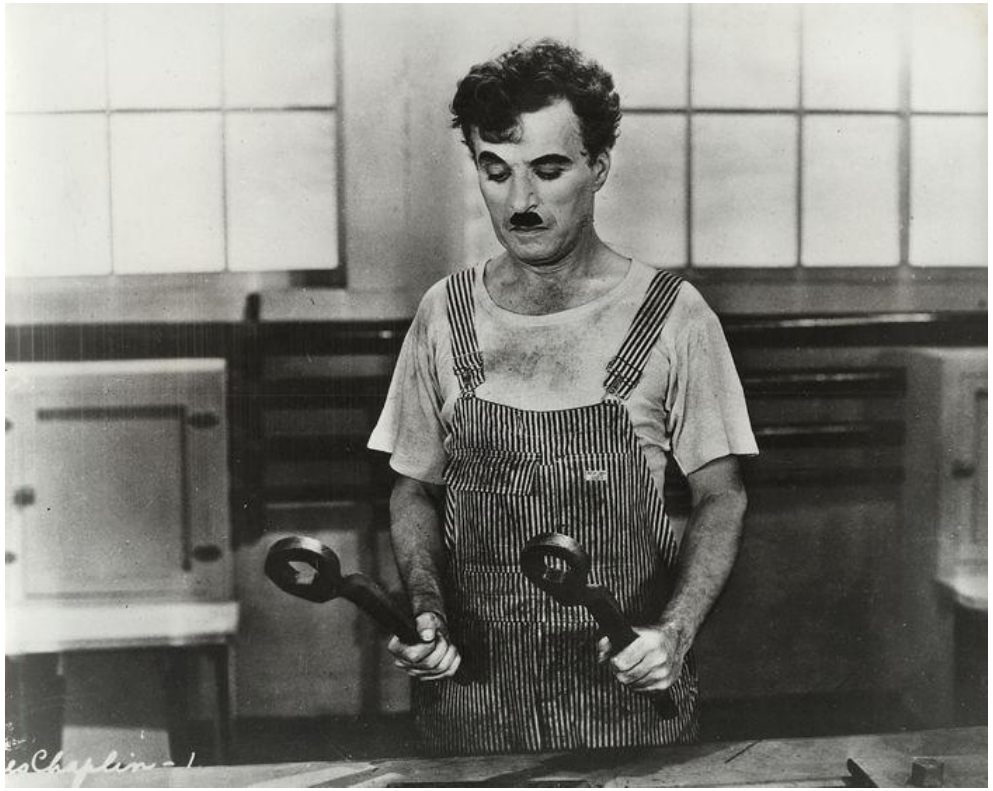
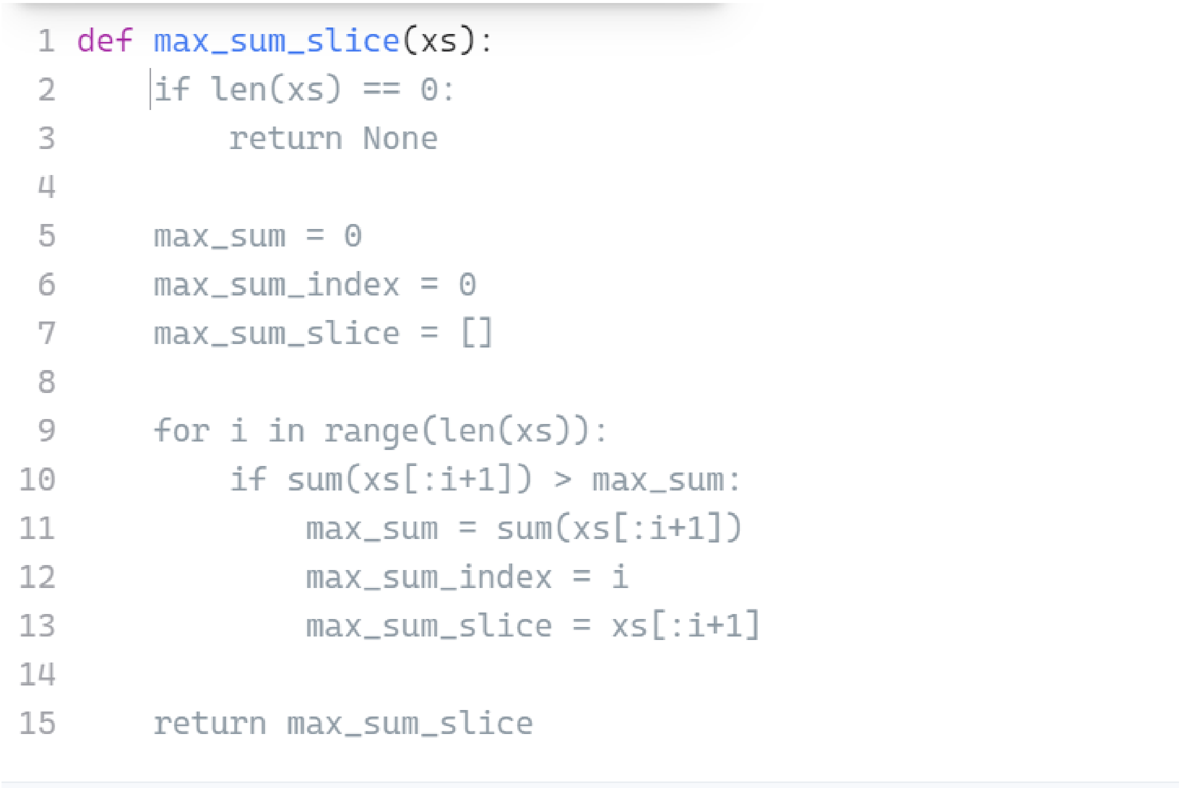
The goal: Automate some programming





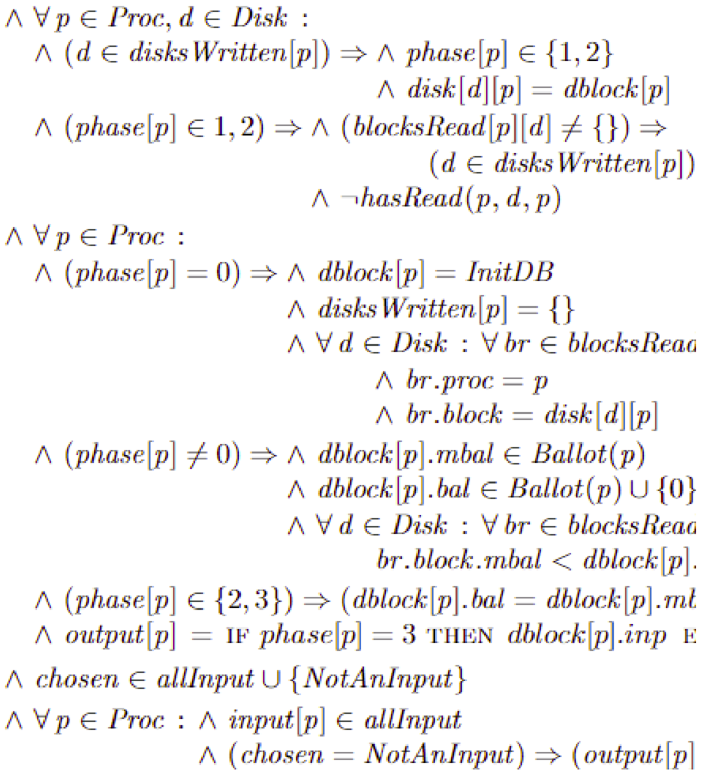
…in a *correct by construction* way

GitHub Copilot



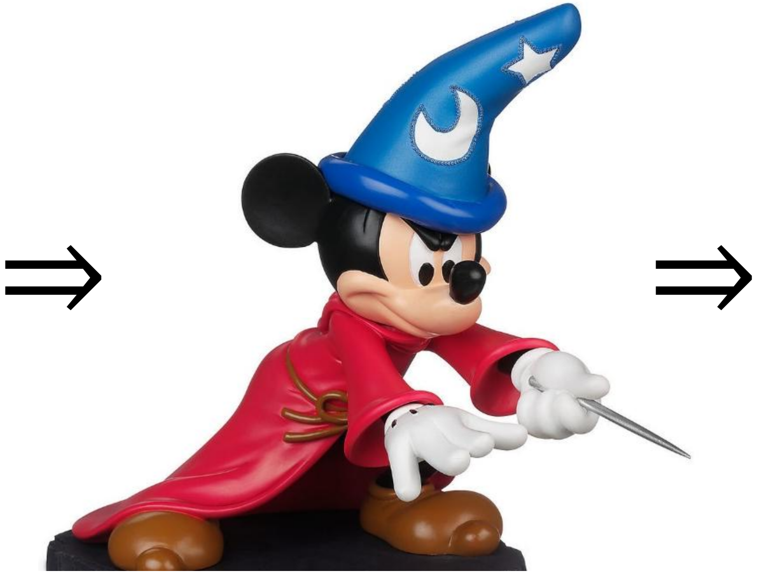
```python
def max_sum_slice(xs):
    if len(xs) == 0:
        return None

    max_sum = 0
    max_sum_index = 0
    max_sum_slice = []

    for i in range(len(xs)):
        if sum(xs[:i+1]) > max_sum:
            max_sum = sum(xs[:i+1])
            max_sum_index = i
            max_sum_slice = xs[:i+1]

    return max_sum_slice
```

Specifications go in, code comes out

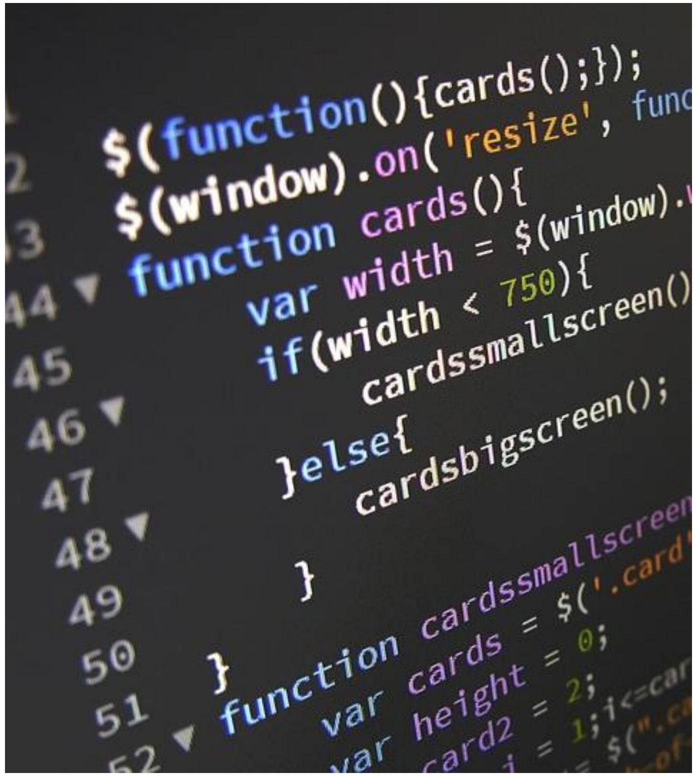


$$
\begin{aligned}
&\land \forall\, p \in \mathit{Proc}, d \in \mathit{Disk} \;:\\
&\quad \land\, (d \in \mathit{disksWritten}[p]) \Rightarrow \land\; \mathit{phase}[p] \in \{1, 2\}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\;\; \land\; \mathit{disk}[d][p] = \mathit{dblock}[p]\\
&\quad \land\, (\mathit{phase}[p] \in 1, 2) \Rightarrow \land\, (\mathit{blocksRead}[p][d] \neq \{\}) \Rightarrow\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad (d \in \mathit{disksWritten}[p])\\
&\qquad\qquad\qquad\qquad\qquad\quad \land\, \neg \mathit{hasRead}(p, d, p)\\
&\land \forall\, p \in \mathit{Proc} \;:\\
&\quad \land\, (\mathit{phase}[p] = 0) \Rightarrow \land\; \mathit{dblock}[p] = \mathit{InitDB}\\
&\qquad\qquad\qquad\qquad\qquad \land\; \mathit{disksWritten}[p] = \{\}\\
&\qquad\qquad\qquad\qquad\qquad \land\, \forall\, d \in \mathit{Disk} \;:\; \forall\, \mathit{br} \in \mathit{blocksReac}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad \land\; \mathit{br.proc} = p\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad \land\; \mathit{br.block} = \mathit{disk}[d][p]\\
&\quad \land\, (\mathit{phase}[p] \neq 0) \Rightarrow \land\; \mathit{dblock}[p].\mathit{mbal} \in \mathit{Ballot}(p)\\
&\qquad\qquad\qquad\qquad\qquad \land\; \mathit{dblock}[p].\mathit{bal} \in \mathit{Ballot}(p) \cup \{0\}\\
&\qquad\qquad\qquad\qquad\qquad \land\, \forall\, d \in \mathit{Disk} \;:\; \forall\, \mathit{br} \in \mathit{blocksReac}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad \mathit{br.block.mbal} < \mathit{dblock}[p].\\
&\quad \land\, (\mathit{phase}[p] \in \{2, 3\}) \Rightarrow (\mathit{dblock}[p].\mathit{bal} = \mathit{dblock}[p].\mathit{ml}\\
&\quad \land\; \mathit{output}[p] \;=\; \textsc{if}\; \mathit{phase}[p] = 3 \;\textsc{then}\; \mathit{dblock}[p].\mathit{inp}\;\; \textsc{e}\\
&\land\; \mathit{chosen} \in \mathit{allInput} \cup \{\mathit{NotAnInput}\}\\
&\land \forall\, p \in \mathit{Proc} \;:\; \land\; \mathit{input}[p] \in \mathit{allInput}\\
&\qquad\qquad\qquad\quad\;\; \land\, (\mathit{chosen} = \mathit{NotAnInput}) \Rightarrow (\mathit{output}[p]
\end{aligned}
$$

⇒



⇒



```
$(function(){cards();});
$(window).on('resize', func
function cards(){
    var width = $(window).
    if(width < 750){
        cardssmallscreen()
    }else{
        cardsbigscreen();
    }
}
function cardssmallscreen
    var cards = $('.card
    var height = 0;
    var card2 = 2;
    i = 1;i<=car
```

What is program synthesis?

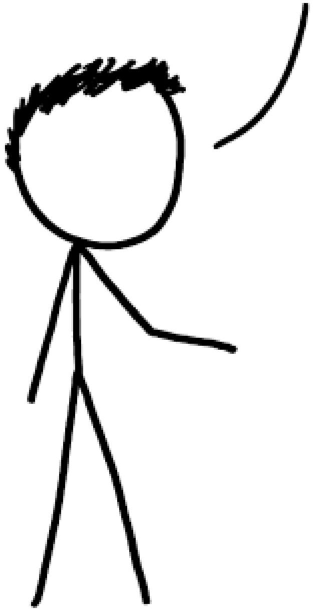
$$\overbrace{\exists p}^{\text{Find a program}} \in \underbrace{\mathcal{L}(G)}_{\text{From a language}}.\overbrace{\forall x.\phi(p,x)}^{\text{A correctness criterion}}$$

How do you find a program?

How do you express correctness?

How do you express correctness?

Specification of intent – the olden days



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\qquad\qquad\qquad\qquad\quad \land \forall d \in \mathit{Disk} : \forall br \in \mathit{blocksRea} \\
\qquad\qquad\qquad\qquad\qquad\quad \land br.\mathit{proc} = p \\
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\quad \land \mathit{output}[p] = \text{IF } \mathit{phase}[p] = 3 \text{ THEN } \mathit{dblock}[p].\mathit{inp} \text{ E} \\
\land \mathit{chosen} \in \mathit{allInput} \cup \{\mathit{NotAnInput}\} \\
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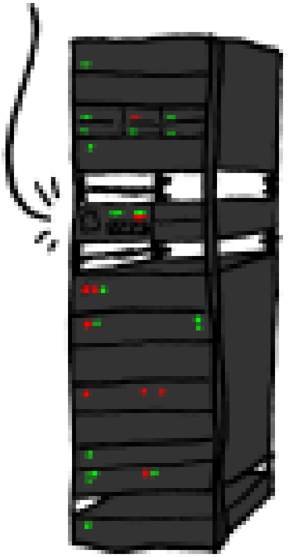
\qquad\qquad\qquad\quad\;\; \land (\mathit{chosen} = \mathit{NotAnInput}) \Rightarrow (\mathit{output}[p]
\end{array}
$$

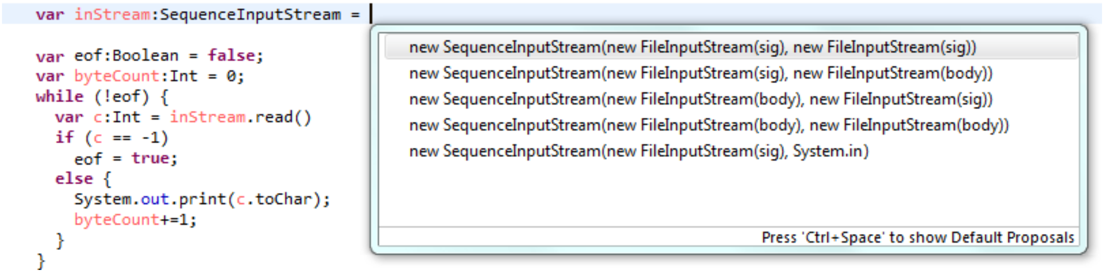
Right-O, here's some code:



Specification of *partial* intent

$$\mathcal{E} = \{\iota_i \rightarrow \omega_i\}$$

Examples



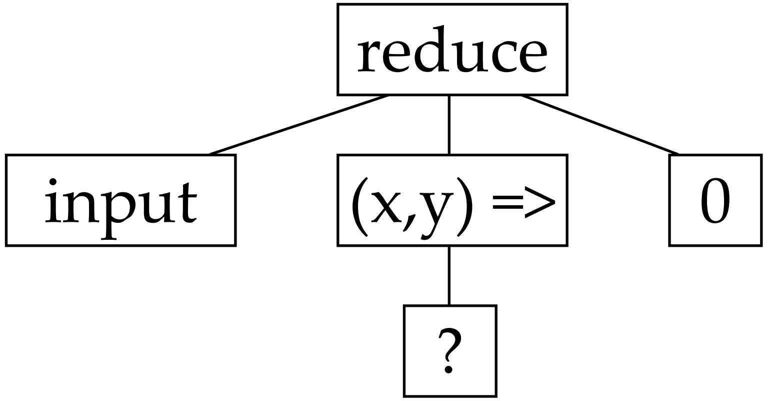
```
var inStream:SequenceInputStream = 

var eof:Boolean = false;
var byteCount:Int = 0;
while (!eof) {
  var c:Int = inStream.read()
  if (c == -1)
    eof = true;
  else {
    System.out.print(c.toChar);
    byteCount+=1;
  }
}
```

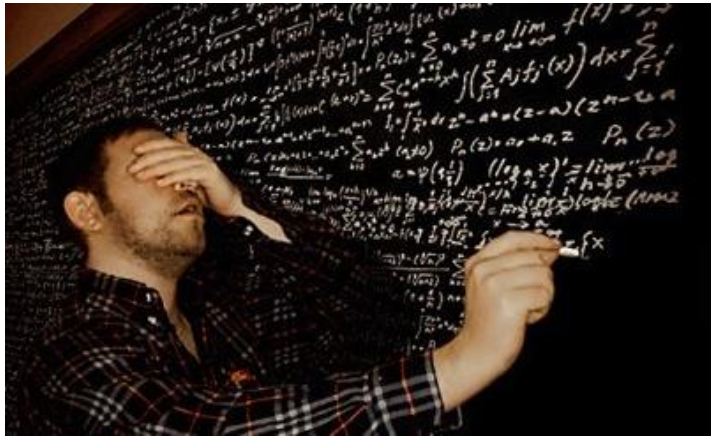
```
new SequenceInputStream(new FileInputStream(sig), new FileInputStream(sig))
new SequenceInputStream(new FileInputStream(sig), new FileInputStream(body))
new SequenceInputStream(new FileInputStream(body), new FileInputStream(sig))
new SequenceInputStream(new FileInputStream(body), new FileInputStream(body))
new SequenceInputStream(new FileInputStream(sig), System.in)
```

Press 'Ctrl+Space' to show Default Proposals

Types



Sketching



Logical specifications

Programming by Example

$$\phi(p, x) = (x = \iota_1 \Rightarrow [\![p]\!](x) = \omega_1) \wedge (x = \iota_2 \Rightarrow [\![p]\!](x) = \omega_2) \wedge \cdots$$

or in other words

$$\mathcal{E} = \{\iota_i \rightarrow \omega_i\}$$

values of all the variables/the
state of the environment

output of an expression/an effect/
the new state of the environment

Type-Driven Synthesis

$$(\tau_1, \tau_2, \ldots, \tau_k) \rightarrow \tau_{out}$$

or in other words,

“Use variables of these types that are in scope to make something of this type”

```
Eq a => a -> [a] -> Maybe [a]
```

(Fancy-)Type-Driven Synthesis

$$(\tau_1, \tau_2, \dots, \tau_k) \rightarrow \tau_{out}$$

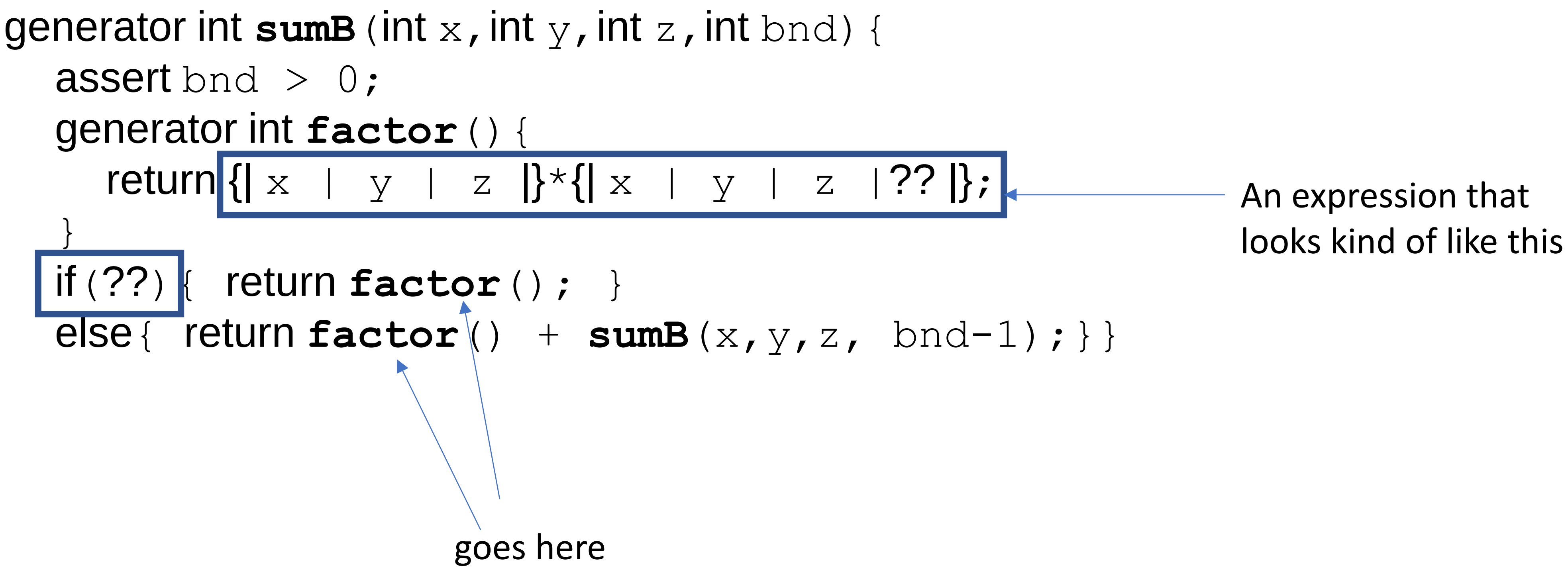
or in other words,

"Use variables of these types that are in scope to make something of this type"

```
n:Nat -> x:a -> {List a | len _v = n}
```

Sketching

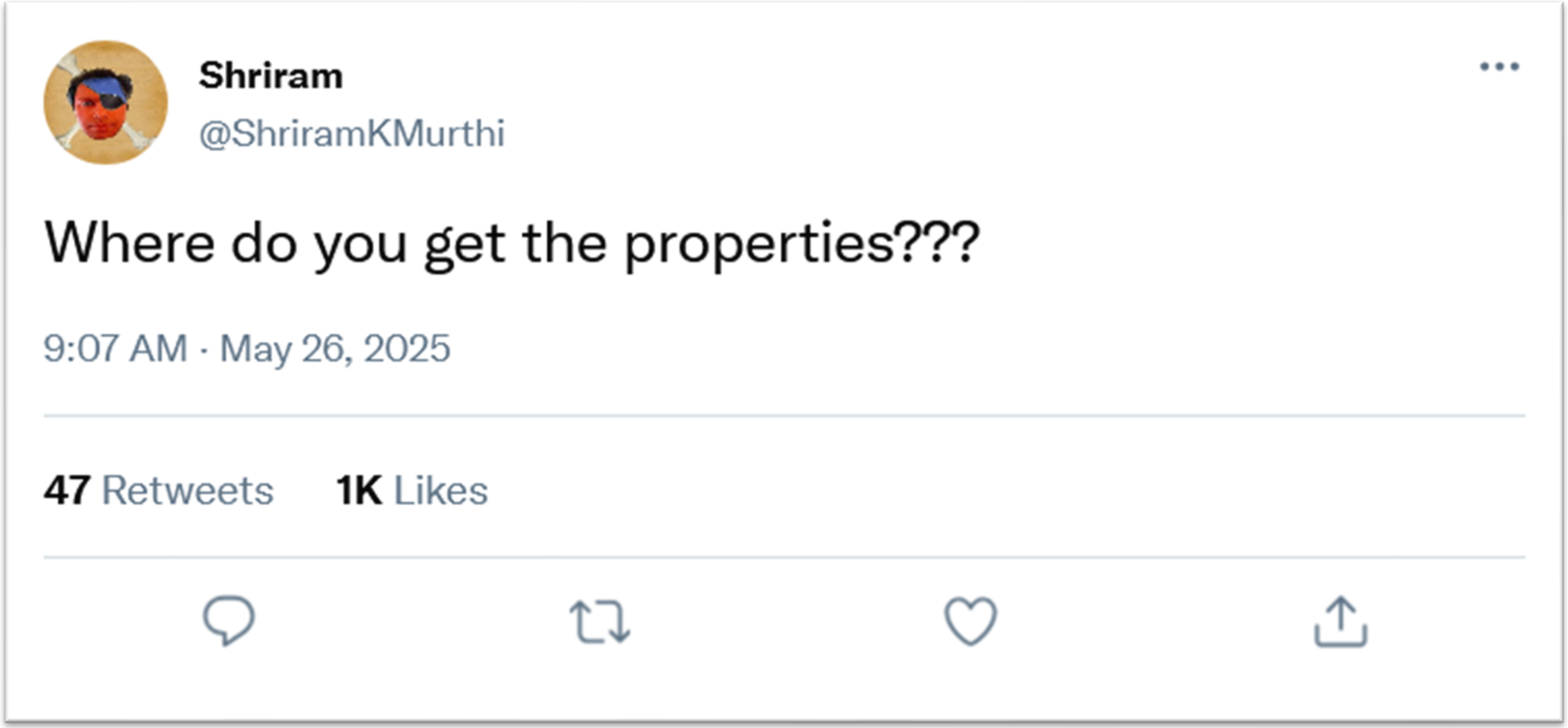
“I already know some of the code for my program”



```
generator int sumB(int x, int y, int z, int bnd) {
    assert bnd > 0;
    generator int factor() {
        return {| x | y | z |}*{| x | y | z |?? |};
    }
    if(??){ return factor(); }
    else{ return factor() + sumB(x,y,z, bnd-1);}}
```

An expression that looks kind of like this

goes here



Shriram
@ShriramKMurthi

Where do you get the properties???

9:07 AM · May 26, 2025

47 Retweets 1K Likes

(next time, I promise)

How do you find a program?

Generic synthesis recipe

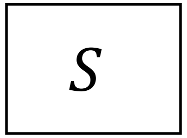
1. Generate a candidate program

- Enumerate trees
  - Top-down
  - Bottom-up
- Traverse automata
- Graph reachability
- Enumerate deduction rules

2. Test against specification

- Run tests
  - Examples
  - Unit
- Encode for SMT solver
- Apply typing rules

Enumerating trees



S

$$S \rightarrow E$$
$$E \rightarrow E + E$$
$$E \rightarrow E\ *\ E$$
$$E \rightarrow [EList]$$
$$EList \rightarrow E \mid EList, E$$
$$E \rightarrow len(E)$$
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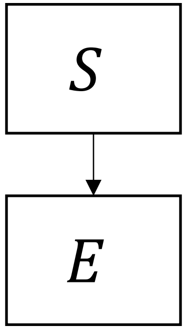
$$E \rightarrow num$$

Enumerating trees



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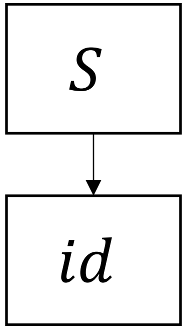

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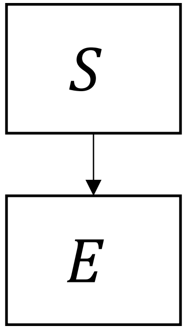

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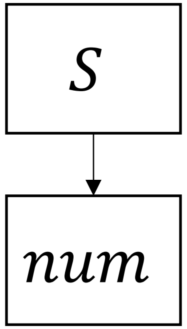

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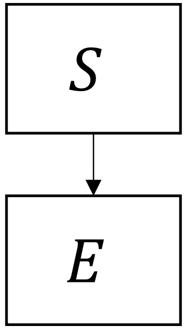

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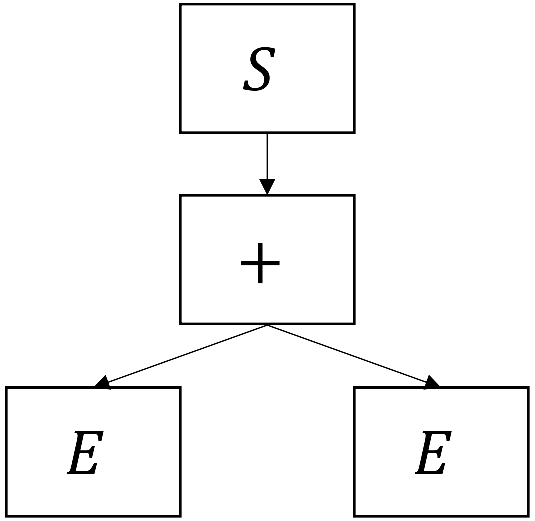

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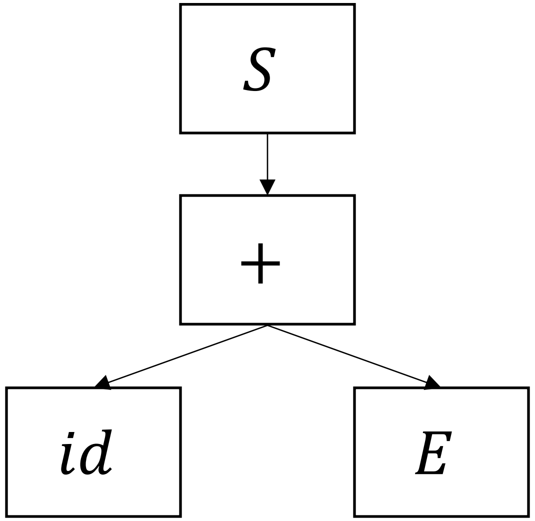

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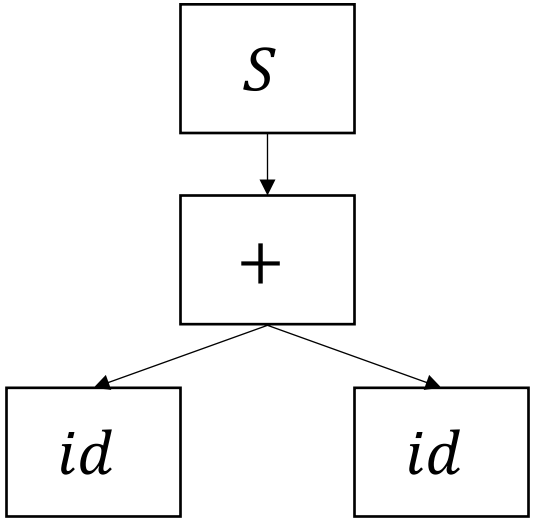

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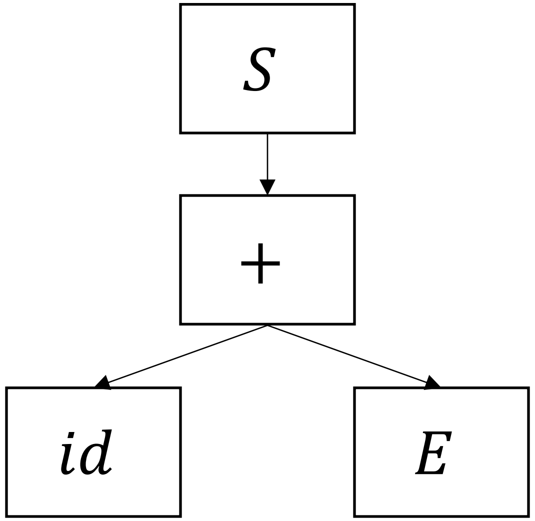

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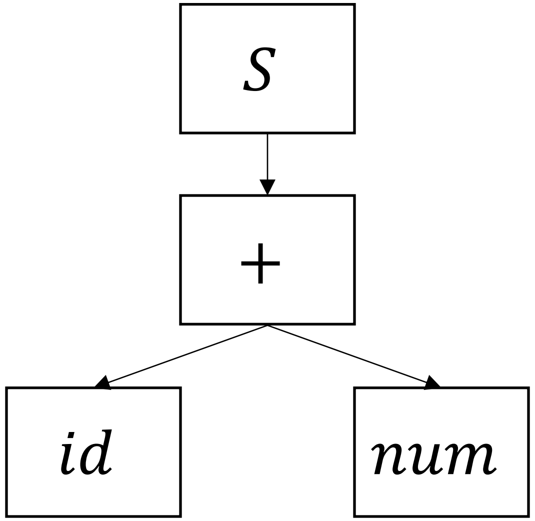

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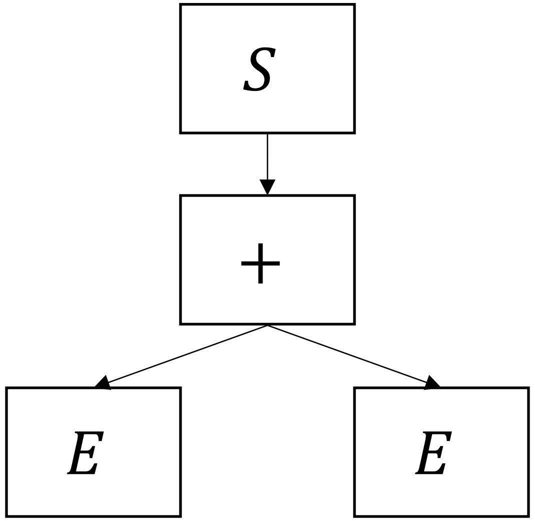

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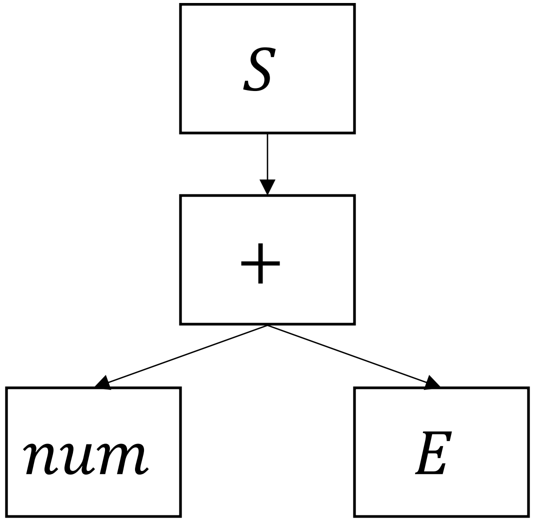

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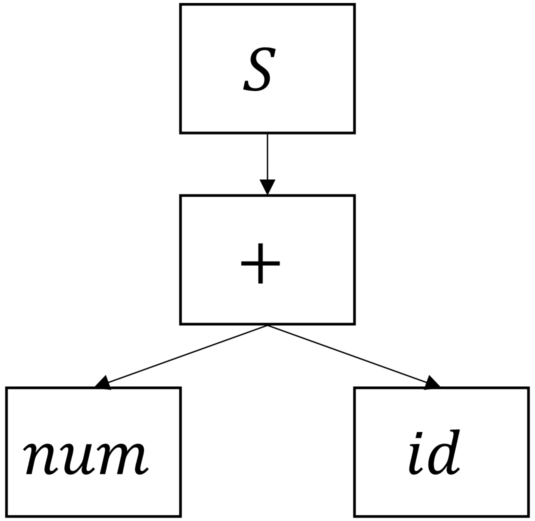

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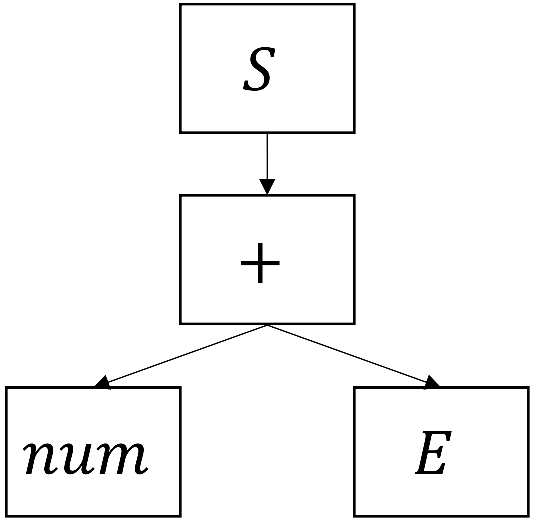

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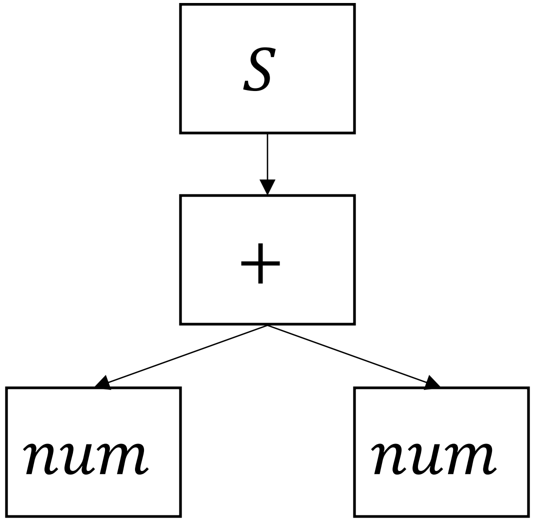

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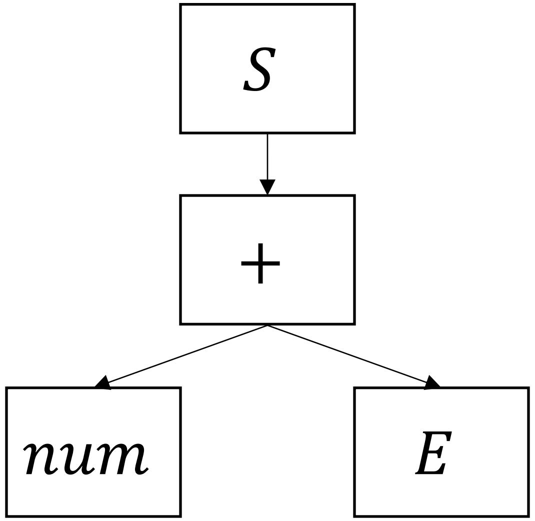

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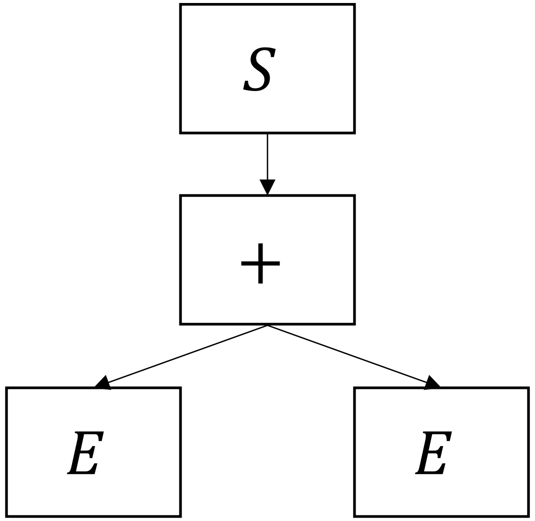

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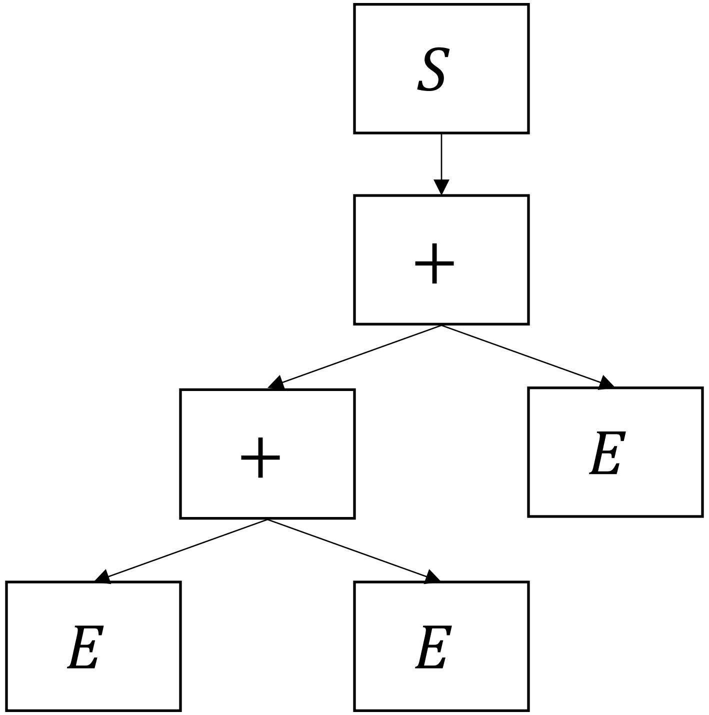

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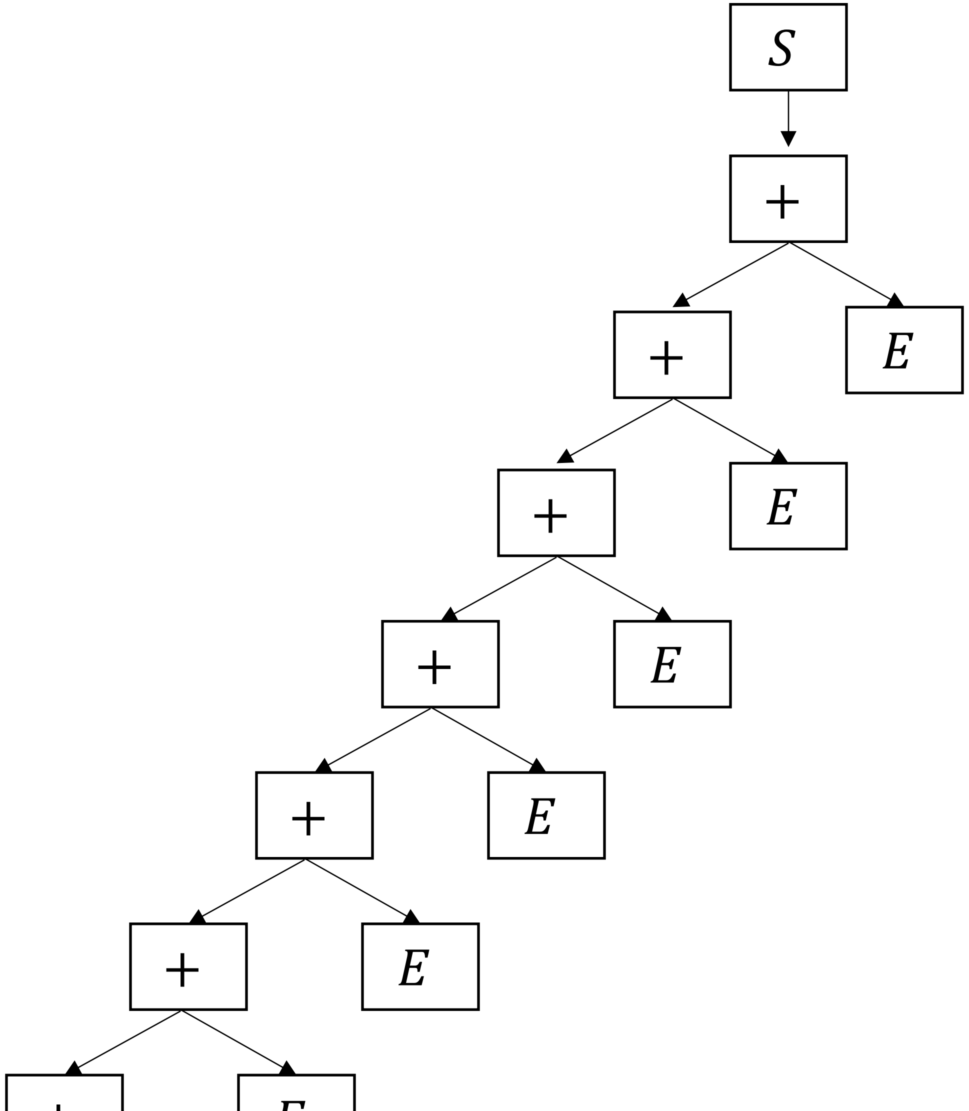
Enumeration stack overflow



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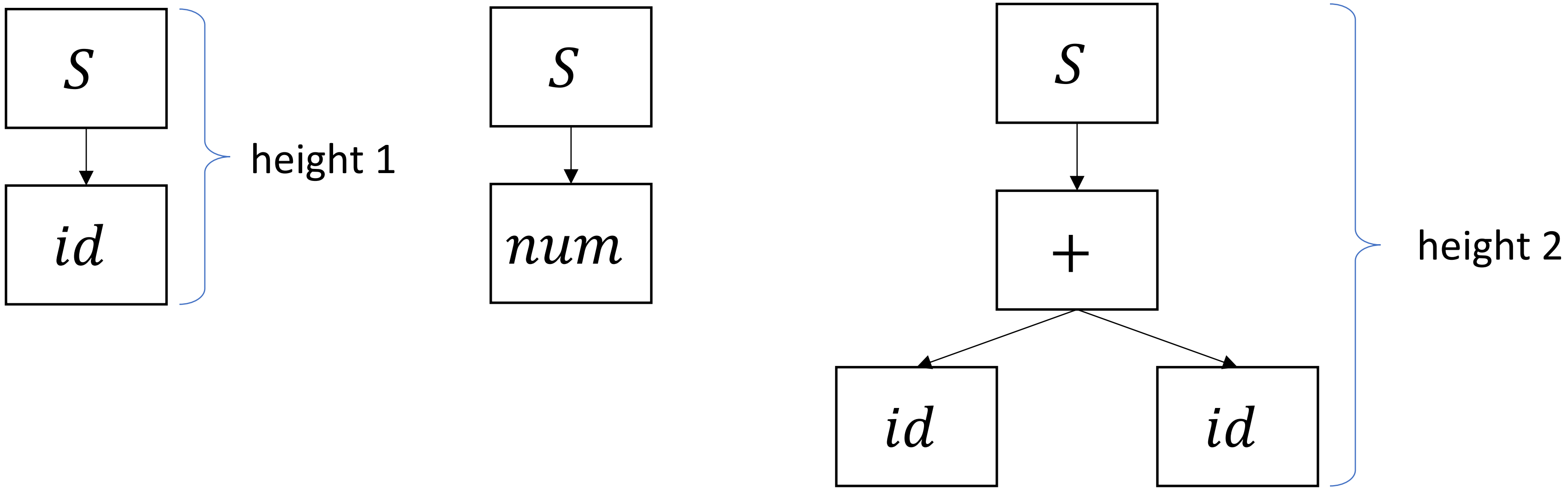
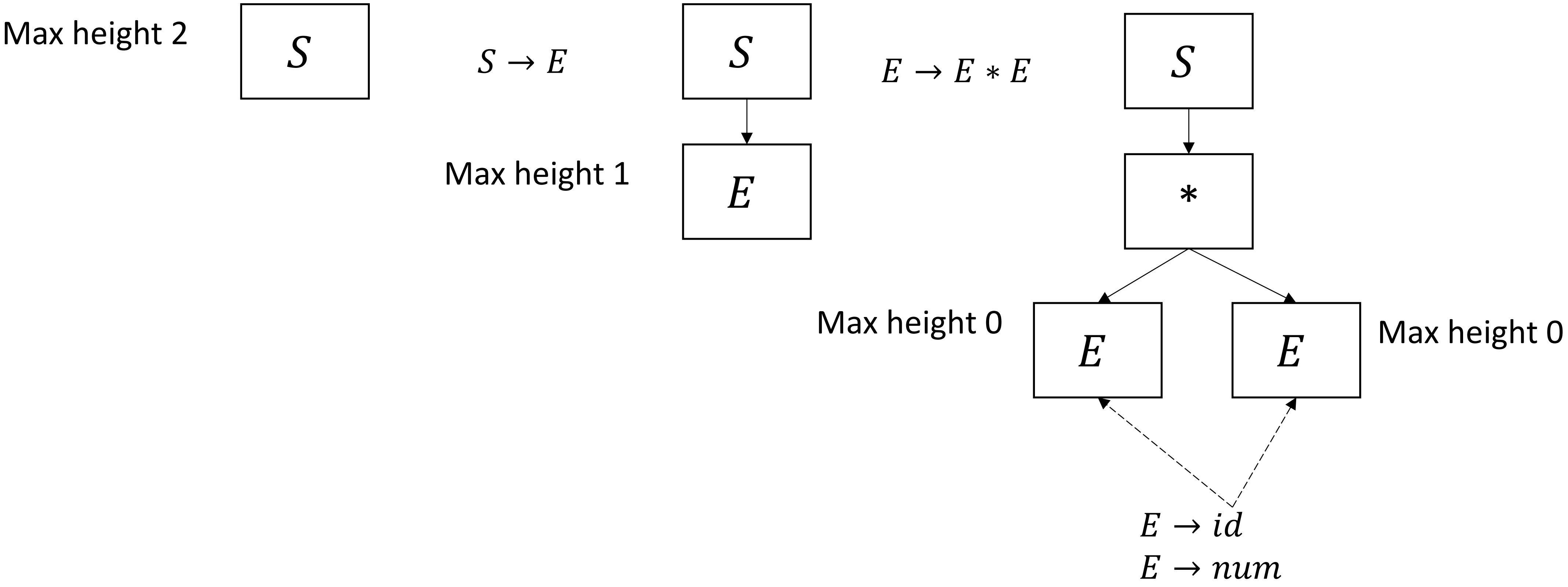
Solution: iterative deepening

- Limit the height of programs being enumerated
- First, all programs of height 0
- then 1
- then 2…

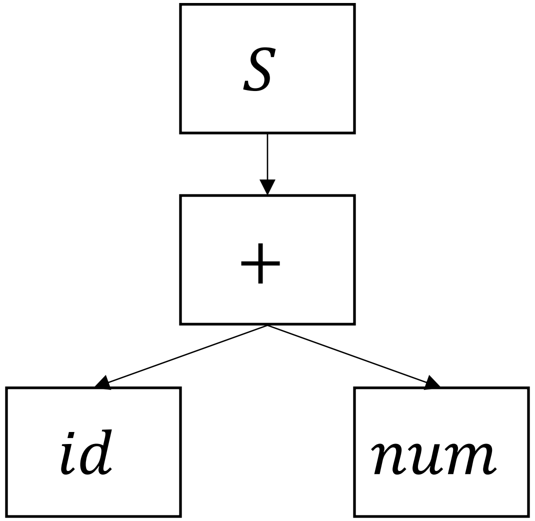


Iterative deepening

To enumerate height 2:

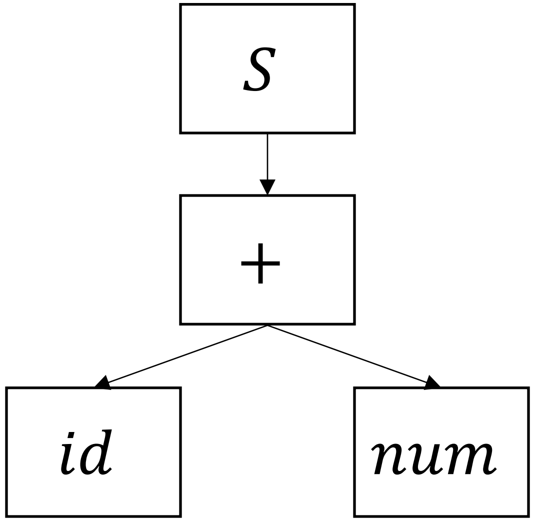


An infinite space

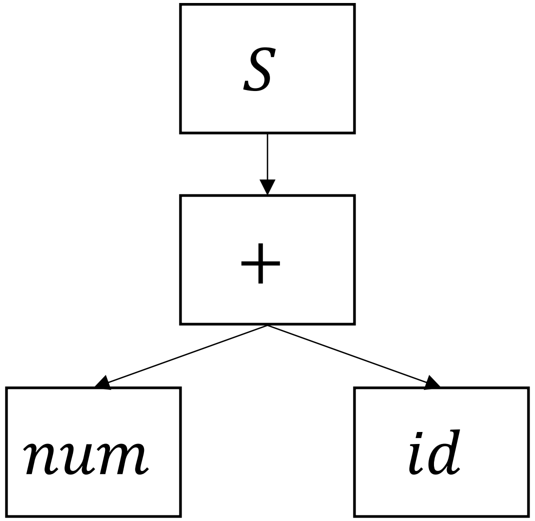


```
x  +  2
x  +  3
x  +  42
y  +  2
y  +  -3
```

An infinite space full of redundancy

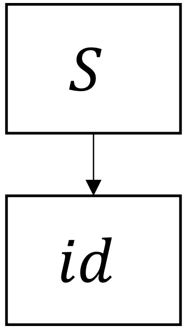


x + 2

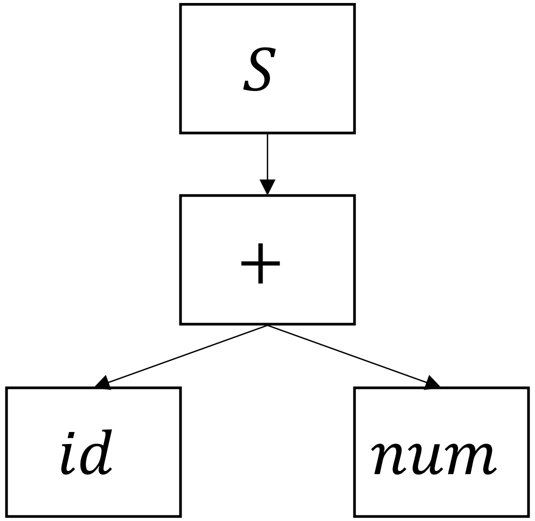


2 + x

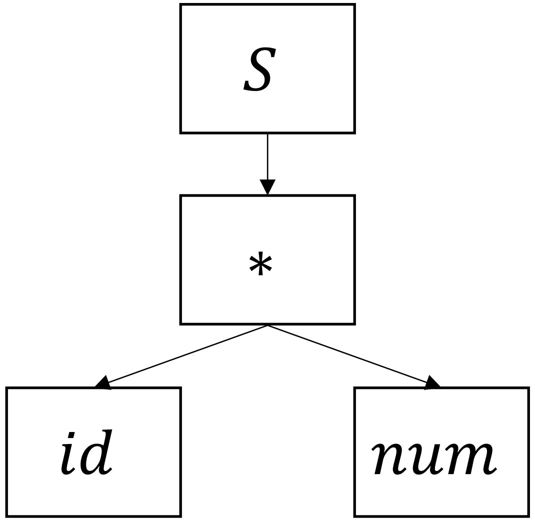
An infinite space full of redundancy



x

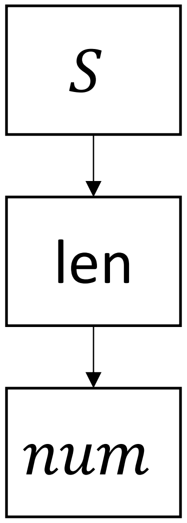


x + 0

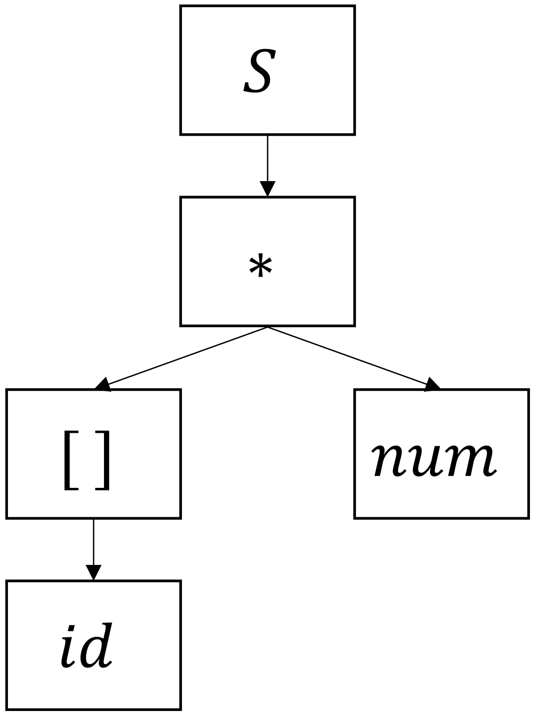


x * 1

An infinite space full of bad programs



```
len(2)
```



```
[x]   *   1
```

Maybe enumerating the space in full is bad, actually?

Pruning the space 1: well-typedness

$$\frac{e_1 : int \quad e_2 : int}{e_1 + e_2 : int} \qquad \frac{e_1 : [\tau] \quad e_2 : [\tau]}{e_1 + e_2 : [\tau]}$$

$$\frac{e_1 : int \quad e_2 : int}{e_1 * e_2 : int} \qquad \frac{e_1, \ldots, e_k : \tau}{[e_1, \ldots, e_k] : [\tau]}$$

$$\frac{e : [\tau]}{len(e) : int} \qquad \frac{var\ x\ has\ type\ \tau}{x : \tau}$$

$$\frac{}{num : int}$$

$$\begin{aligned}
&S \rightarrow E \\
&E \rightarrow E + E \\
&E \rightarrow E\ *\ E \\
&E \rightarrow [EList] \\
&EList \rightarrow \varepsilon \mid EList, E \\
&E \rightarrow len(E) \\
&E \rightarrow id \\
&E \rightarrow num
\end{aligned}$$

Generic synthesis recipe

1. Generate a candidate program

- Enumerate trees
  - Top-down
  - Bottom-up
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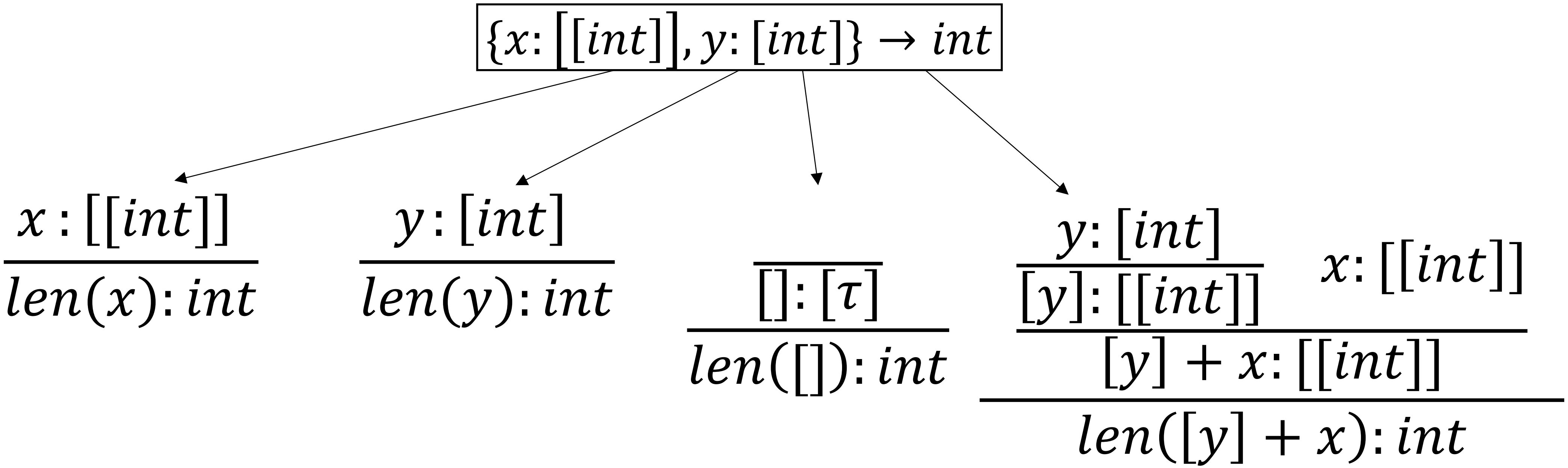
Looking at spec with types



$$\boxed{\{x: [[int]], y: [int]\} \to int}$$

$$\frac{x : [[int]]}{len(x): int}$$

$$\frac{y: [int]}{len(y): int}$$

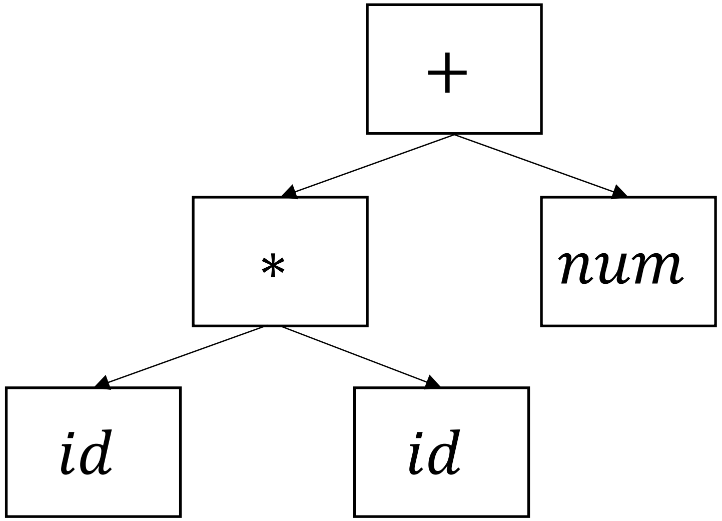
$$\frac{\overline{[]: [\tau]}}{len([]): int}$$

$$\frac{\dfrac{\dfrac{y: [int]}{[y]: [[int]]} \qquad x: [[int]]}{[y] + x: [[int]]}}{len([y] + x): int}$$

Deductive proof search

What about examples?

What we enumerated was not a concrete program



Can we combine that with examples?

What about examples?

What we enumerated was not a concrete program

Given:

$$\epsilon_1\text{: } \{x \mapsto 6, y \mapsto 8\} \to 49$$
$$\epsilon_2\text{:} \{x \mapsto 4, y \mapsto 7\} \to 29$$

And a (meta) candidate: `(id`$_1$` * id`$_2$`) + num`

Can we solve for id_1, id_2, num?

SMT solvers to the rescue

Given:

$$\epsilon_1\colon \{x \mapsto 6, y \mapsto 8\} \to 49$$
$$\epsilon_2\colon \{x \mapsto 4, y \mapsto 7\} \to 29$$

And a (meta) candidate: $(\texttt{id}_1\ *\ \texttt{id}_2)\ +\ \texttt{num}$

Can we solve for id_1, id_2, num?

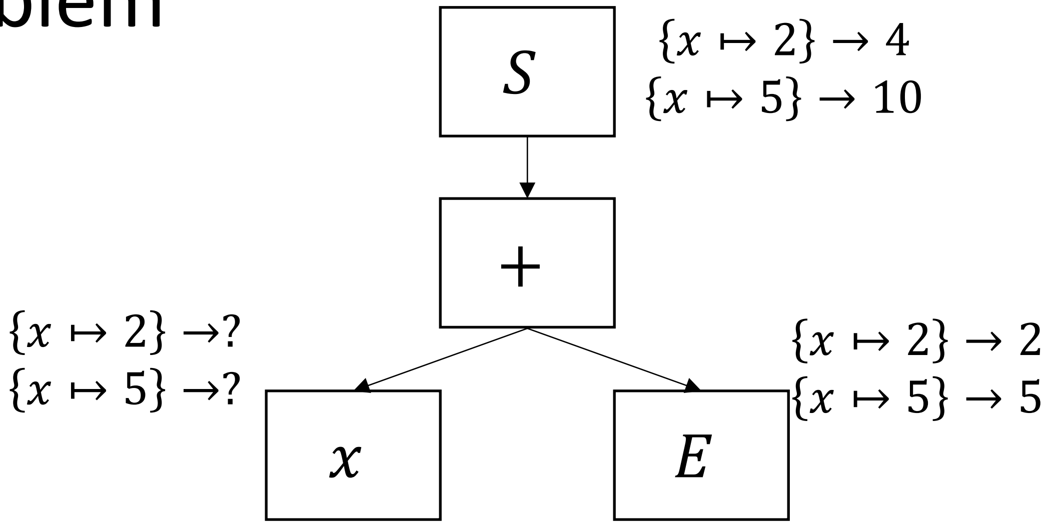
$$\forall arr \in \overset{x}{\overset{\downarrow}{\mathbb{Z}}} \times \overset{y}{\overset{\downarrow}{\mathbb{Z}}}. (arr = [6{,}8] \Rightarrow (arr[i_1] * arr[i_2]) + num = 49) \wedge$$
$$(arr = [4{,}7] \Rightarrow (arr[i_1] * arr[i_2]) + num = 29)$$

Model: $i_1 = 1, i_2 = 0, num = 1$

free variables

Can we do better?

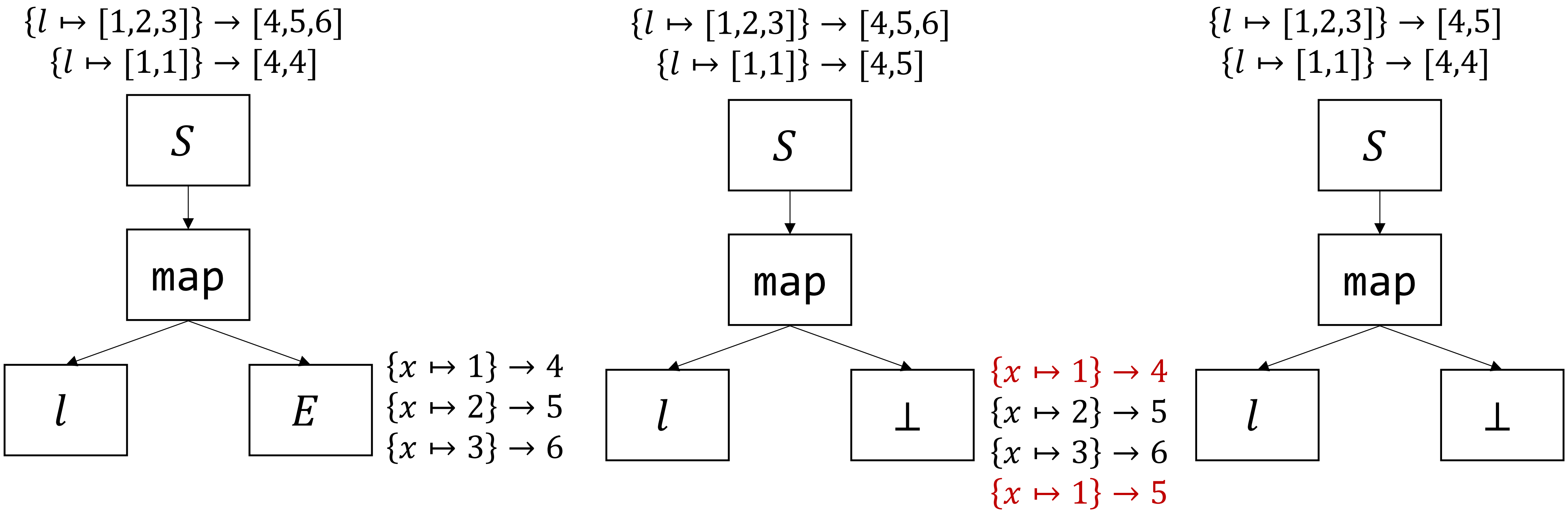
- Specification x Semantics of node = Specifications for remaining problem
  - … sometimes



S
$\{x \mapsto 2\} \rightarrow 4$
$\{x \mapsto 5\} \rightarrow 10$
$+$
$\{x \mapsto 2\} \rightarrow ?$
$\{x \mapsto 5\} \rightarrow ?$
x
E
$\{x \mapsto 2\} \rightarrow 2$
$\{x \mapsto 5\} \rightarrow 5$

- Called *Example Refinement*

Refinements and higher-order functions



$\{l \mapsto [1,2,3]\} \rightarrow [4,5,6]$
$\{l \mapsto [1,1]\} \rightarrow [4,4]$

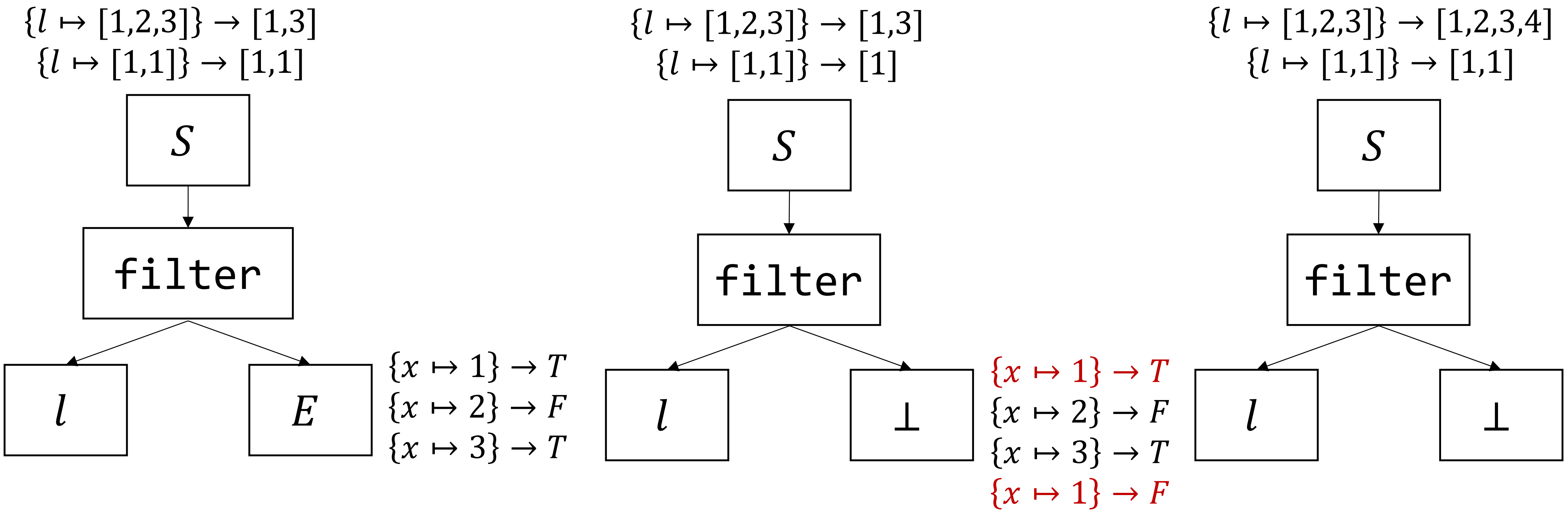
$\{x \mapsto 1\} \rightarrow 4$
$\{x \mapsto 2\} \rightarrow 5$
$\{x \mapsto 3\} \rightarrow 6$

$\{l \mapsto [1,2,3]\} \rightarrow [4,5,6]$
$\{l \mapsto [1,1]\} \rightarrow [4,5]$

$\{x \mapsto 1\} \rightarrow 4$
$\{x \mapsto 2\} \rightarrow 5$
$\{x \mapsto 3\} \rightarrow 6$
$\{x \mapsto 1\} \rightarrow 5$

$\{l \mapsto [1,2,3]\} \rightarrow [4,5]$
$\{l \mapsto [1,1]\} \rightarrow [4,4]$

Refinements and higher-order functions



$\{l \mapsto [1,2,3]\} \rightarrow [1,3]$

$\{l \mapsto [1,1]\} \rightarrow [1,1]$

S

filter

l

E

$\{x \mapsto 1\} \rightarrow T$

$\{x \mapsto 2\} \rightarrow F$

$\{x \mapsto 3\} \rightarrow T$

$\{l \mapsto [1,2,3]\} \rightarrow [1,3]$

$\{l \mapsto [1,1]\} \rightarrow [1]$

S

filter

l

$\bot$

$\{x \mapsto 1\} \rightarrow T$

$\{x \mapsto 2\} \rightarrow F$

$\{x \mapsto 3\} \rightarrow T$

$\{x \mapsto 1\} \rightarrow F$

$\{l \mapsto [1,2,3]\} \rightarrow [1,2,3,4]$

$\{l \mapsto [1,1]\} \rightarrow [1,1]$

S

filter

l

$\bot$

Generic synthesis recipe

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Enumerative Search

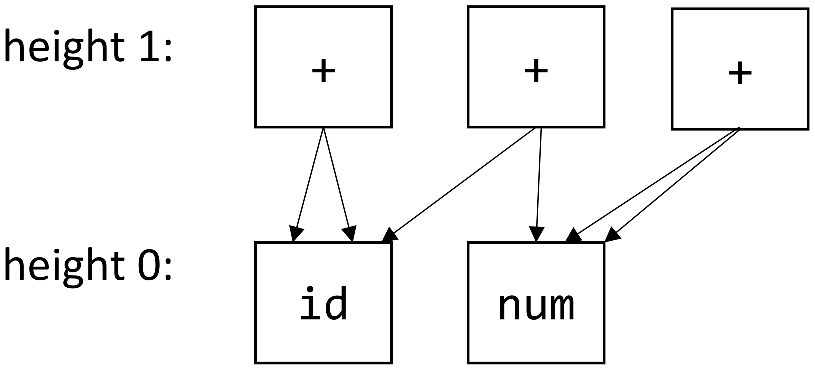
We did great with: Generate programs from the grammar, one by one, and test them on the specification

Enumerating trees

$$S \rightarrow E$$
$$E \rightarrow E + E$$
$$E \rightarrow E \;*\; E$$
$$E \rightarrow [EList]$$
$$EList \rightarrow E \mid EList, E$$
$$E \rightarrow len(E)$$
$$E \rightarrow id$$
$$E \rightarrow num$$

height 0: `id` `num`

Enumerating trees



$S \to E$

$\boxed{E \to E + E}$

$E \to E \, * \, E$

$E \to [EList]$

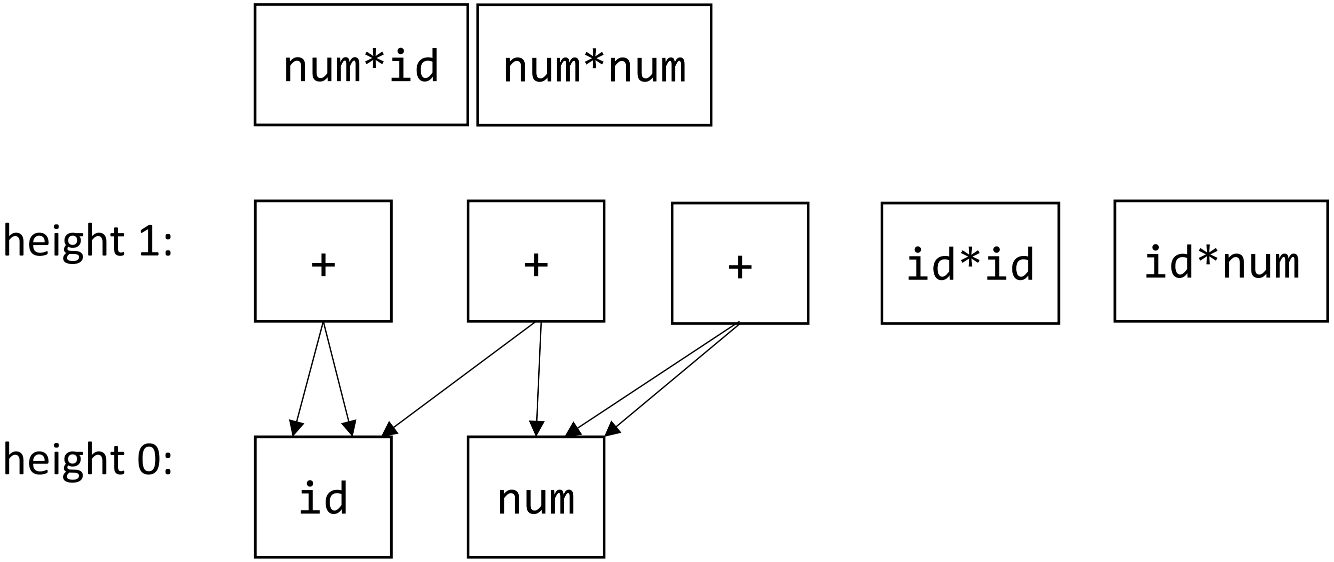
$EList \to E \mid EList, E$

$E \to len(E)$

$E \to id$

$E \to num$

Enumerating trees



$S \rightarrow E$

$E \rightarrow E + E$

$\boxed{E \rightarrow E \; * \; E}$

$E \rightarrow [EList]$

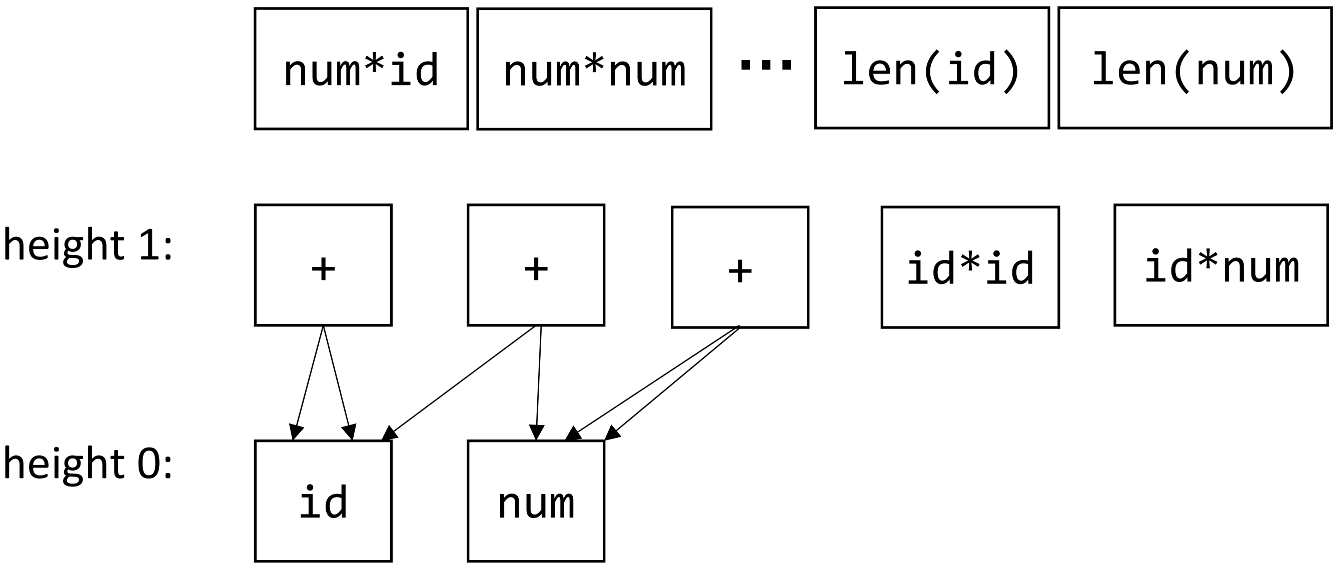
$EList \rightarrow E \mid EList, E$

$E \rightarrow len(E)$

$E \rightarrow id$

$E \rightarrow num$

Enumerating trees



$S \rightarrow E$

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow [EList]$

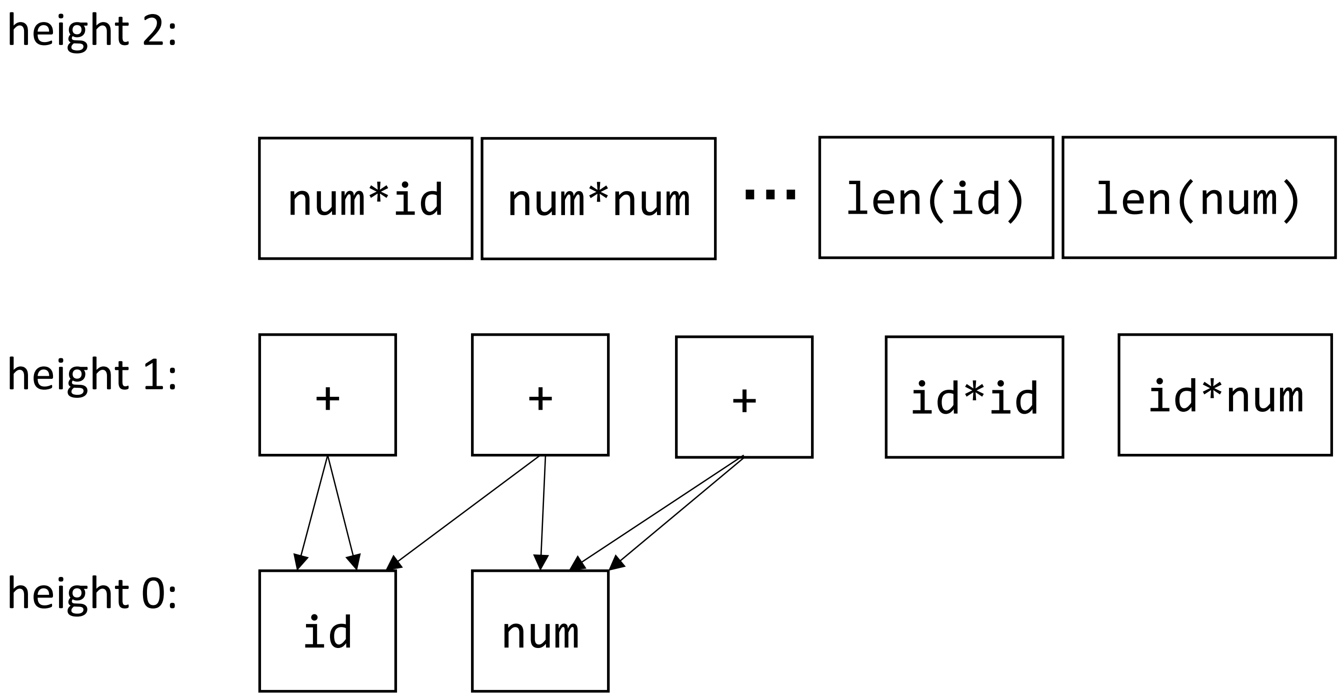
$EList \rightarrow E \mid EList, E$

$\boxed{E \rightarrow len(E)}$

$E \rightarrow id$

$E \rightarrow num$

Enumerating trees



$S \rightarrow E$

$\boxed{E \rightarrow E + E}$

$E \rightarrow E \ * \ E$

$E \rightarrow [EList]$

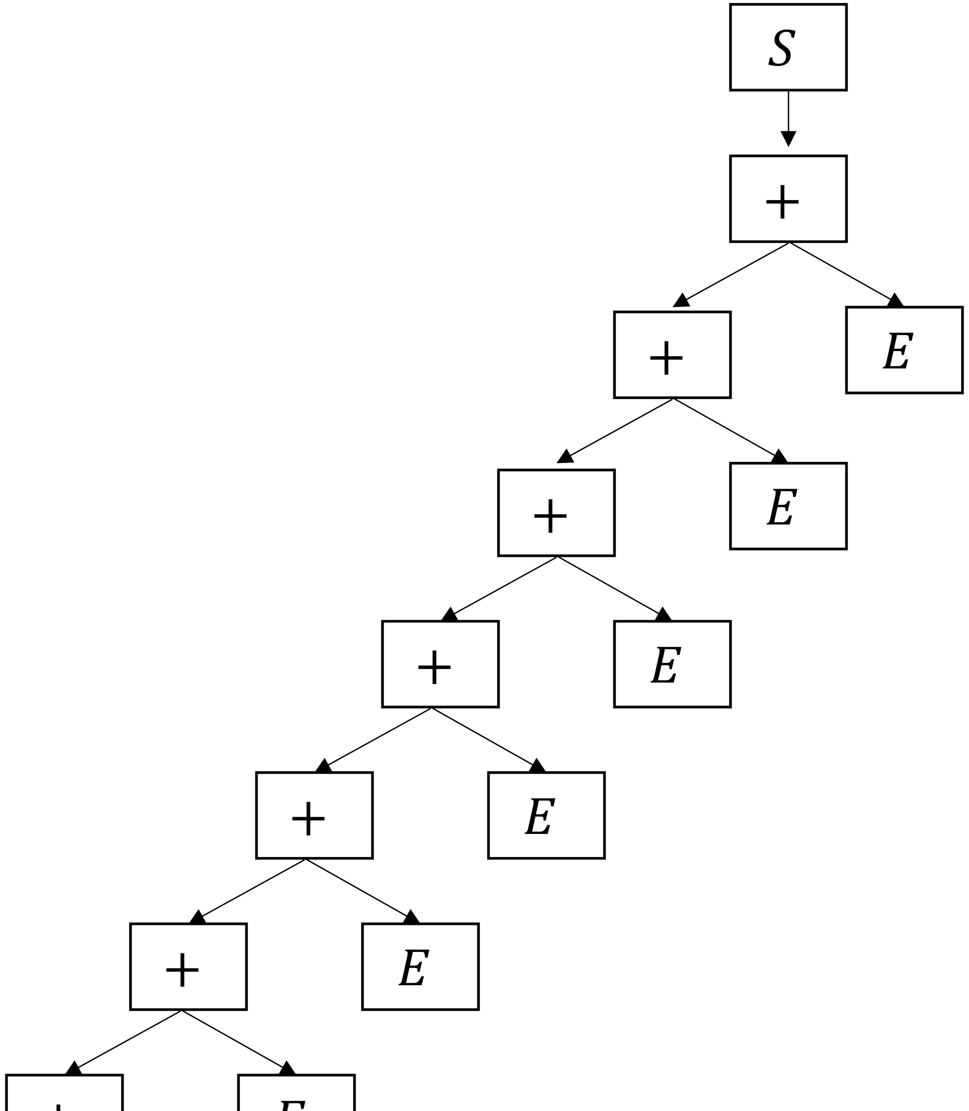
$EList \rightarrow E \mid EList, E$

$E \rightarrow len(E)$

$E \rightarrow id$

$E \rightarrow num$

On one hand: no stack overflow



$S \rightarrow E$
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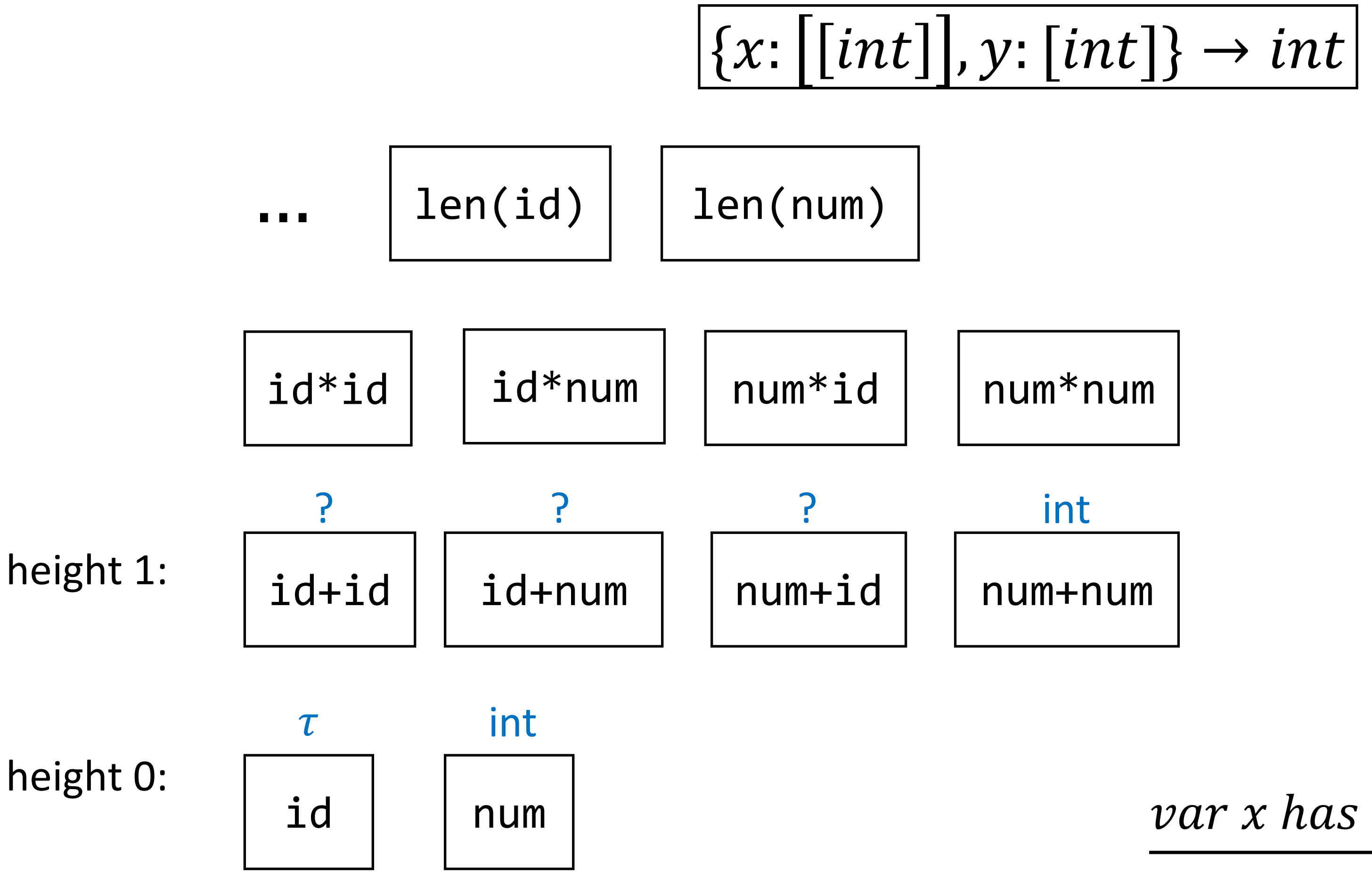
$E \rightarrow E \, * \, E$
$E \rightarrow [EList]$
$EList \rightarrow E \mid EList, E$
$E \rightarrow len(E)$
$E \rightarrow id$
$E \rightarrow num$

On the other hand:

A lot of things suddenly got harder

What if we have a type specification?



$\{x: [[int]], y: [int]\} \rightarrow int$

...

`len(id)` `len(num)`

`id*id` `id*num` `num*id` `num*num`

height 1:

?	?	?	int
`id+id`	`id+num`	`num+id`	`num+num`

height 0:

τ	int
`id`	`num`

$$S \rightarrow E$$
$$E \rightarrow E + E$$
$$E \rightarrow E * E$$
$$E \rightarrow [EList]$$
$$EList \rightarrow E \mid EList, E$$
$$E \rightarrow len(E)$$
$$E \rightarrow id$$
$$E \rightarrow num$$

$$\frac{var\ x\ has\ type\ \tau}{x: \tau} \qquad \frac{}{num: int} \qquad \frac{e_1: int \quad e_2: int}{e_1 + e_2: int}$$

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Programming by Example

$$\phi(p,x) = (x = \iota_1 \Rightarrow [\![p]\!](x) = \omega_1) \wedge (x = \iota_2 \Rightarrow [\![p]\!](x) = \omega_2) \wedge \cdots$$

or in other words

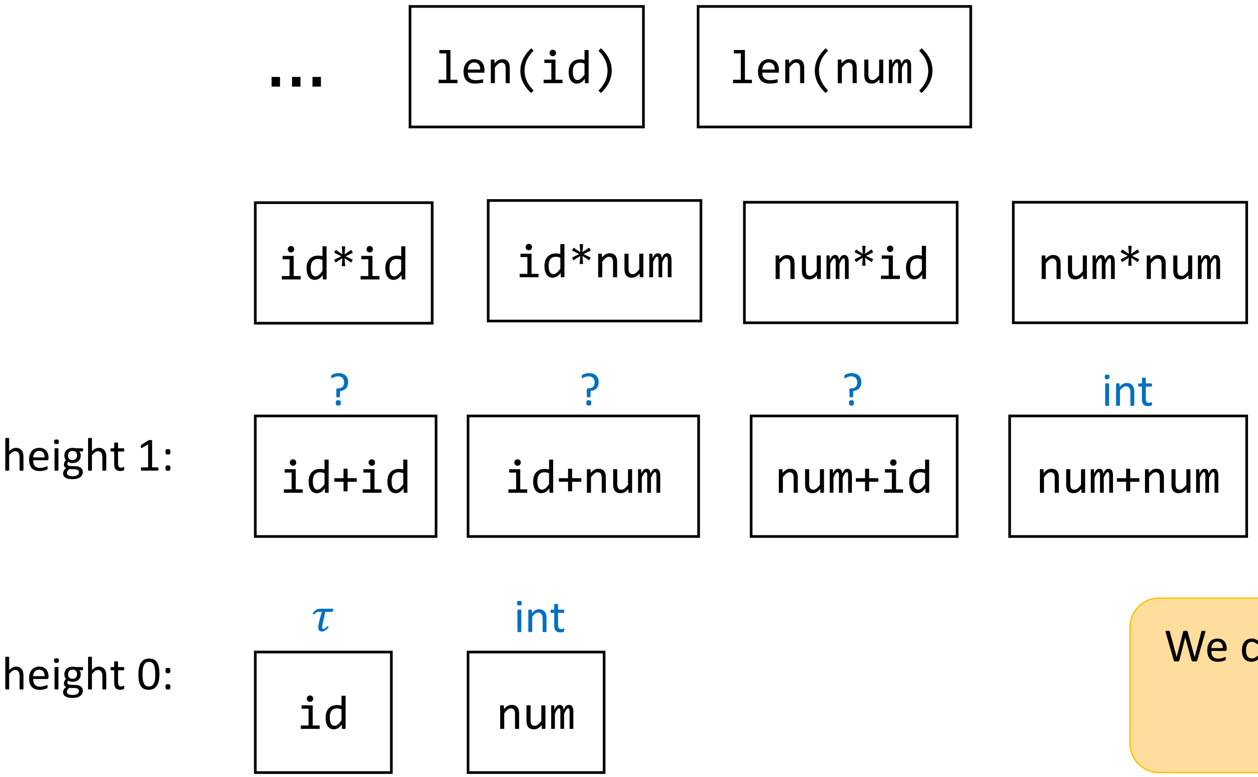
$$\mathcal{E} = \{\iota_i \rightarrow \omega_i\}$$

values of all the variables/the
state of the environment

output of an expression/an effect/
the new state of the environment

Programming by Example

$$\epsilon_1\text{: } \{x \mapsto 6, y \mapsto 8\} \rightarrow 49$$
$$\epsilon_2\text{: } \{x \mapsto 4, y \mapsto 7\} \rightarrow 29$$



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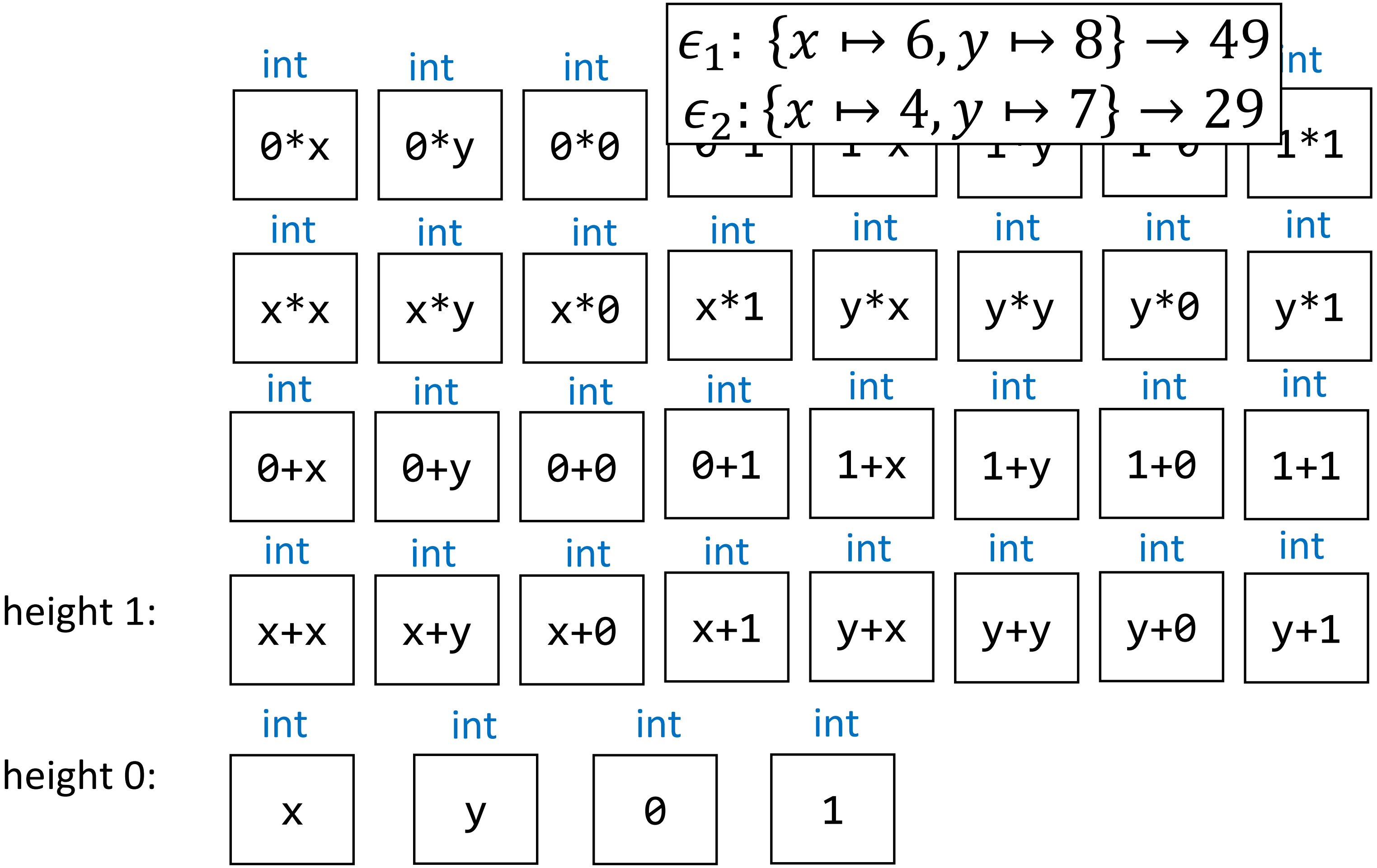
$$E \rightarrow len(E)$$
$$E \rightarrow id$$
$$E \rightarrow num$$

We can evaluate every program!

…almost

Programming by Example

ϵ_1: $\{x \mapsto 6, y \mapsto 8\} \rightarrow 49$
ϵ_2: $\{x \mapsto 4, y \mapsto 7\} \rightarrow 29$



int 0*x | int 0*y | int 0*0 | 0*1 | 1*x | 1*y | 1*0 | int 1*1

int x*x | int x*y | int x*0 | int x*1 | int y*x | int y*y | int y*0 | int y*1

int 0+x | int 0+y | int 0+0 | int 0+1 | int 1+x | int 1+y | int 1+0 | int 1+1

height 1: int x+x | int x+y | int x+0 | int x+1 | int y+x | int y+y | int y+0 | int y+1

height 0: int x | int y | int 0 | int 1

$$S \rightarrow E$$
$$E \rightarrow E + E$$
$$E \rightarrow E \; * \; E$$
$$E \rightarrow [EList]$$
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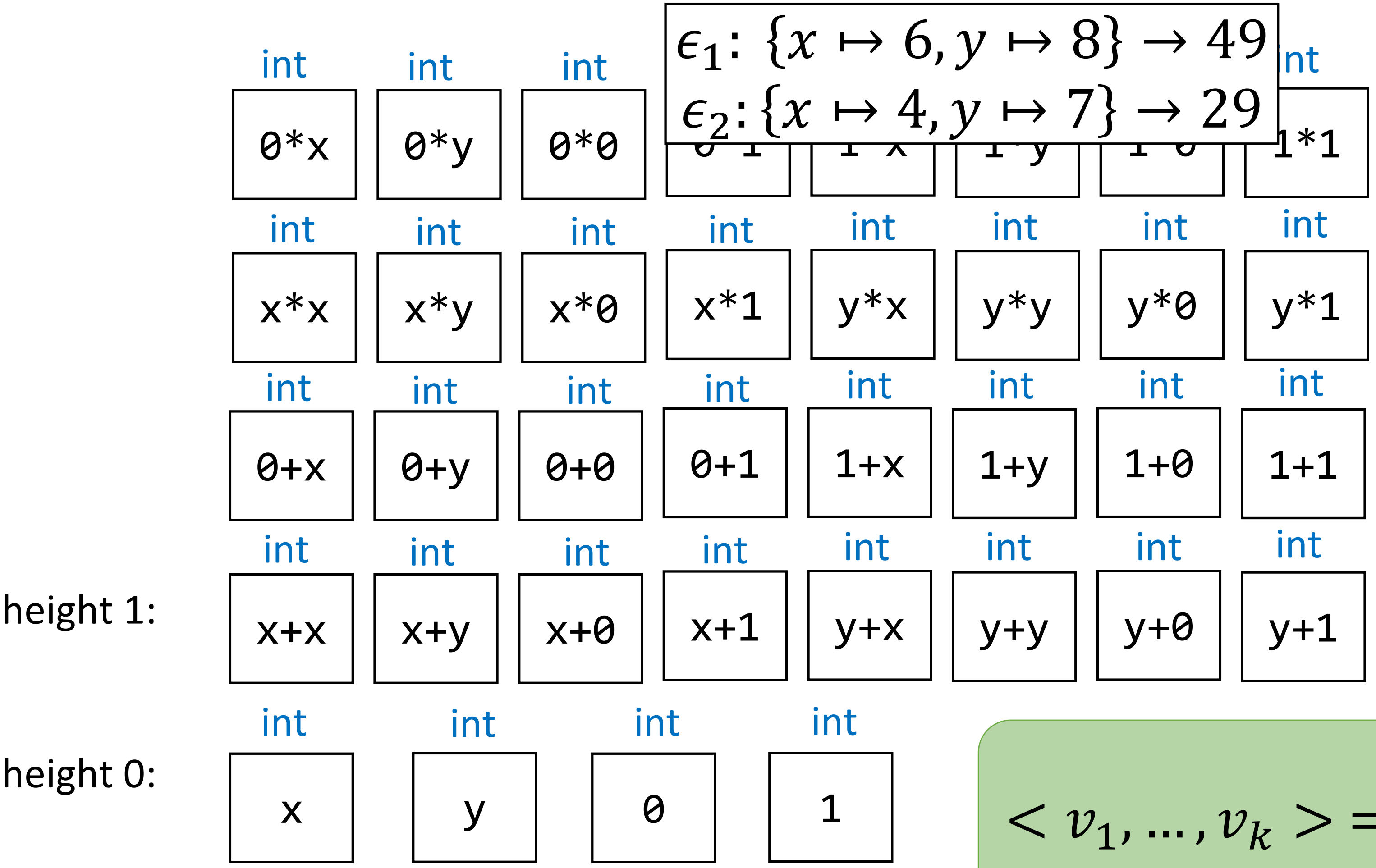
Programming by Example

ϵ_1: $\{x \mapsto 6, y \mapsto 8\} \to 49$
ϵ_2: $\{x \mapsto 4, y \mapsto 7\} \to 29$



int 0*x | int 0*y | int 0*0 | 0*1 | 1*x | 1*y | 1*0 | int 1*1

int x*x | int x*y | int x*0 | int x*1 | int y*x | int y*y | int y*0 | int y*1

int 0+x | int 0+y | int 0+0 | int 0+1 | int 1+x | int 1+y | int 1+0 | int 1+1

height 1: int x+x | int x+y | int x+0 | int x+1 | int y+x | int y+y | int y+0 | int y+1

height 0: int x | int y | int 0 | int 1

$$S \to E$$
$$E \to E + E$$
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$$E \to len(E)$$
$$E \to x \mid y$$
$$E \to 0 \mid 1$$

$$< v_1, \dots, v_k > = < [\![p]\!](\iota_1), \dots, [\![p]\!](\iota_k) >$$

Programming by Example

ϵ_1: $\{x \mapsto 6, y \mapsto 8\} \rightarrow 49$
ϵ_2: $\{x \mapsto 4, y \mapsto 7\} \rightarrow 29$

	<0,0>	<0,0>	<0,0>					1,1>
	0*x	0*y	0*0	0*1	1*x	1*y	1*0	1*1
	<36,16>	<48,28>	<0,0>	<6,4>	<48,28>	<64,49>	<0,0>	<8,7>
	x*x	x*y	x*0	x*1	y*x	y*y	y*0	y*1
	<6,4>	<8,7>	<0,0>	<1,1>	<7,5>	<9,8>	<1,1>	<2,2>
	0+x	0+y	0+0	0+1	1+x	1+y	1+0	1+1
	<12,8>	<14,11>	<6,4>	<7,5>	<14,11>	<16,14>	<8,7>	<9,8>
height 1:	x+x	x+y	x+0	x+1	y+x	y+y	y+0	y+1
	<6,4>	<8,7>	<0,0>	<1,1>				
height 0:	x	y	0	1				

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Equivalence classes

[Albarghouthi et al. 2013, Udupa et al. 2013]

ϵ_1: $\{x \mapsto 6, y \mapsto 8\} \to 49$

ϵ_2: $\{x \mapsto 4, y \mapsto 7\} \to 29$

	<0,0>	<0,0>	<0,0>					1,1>
	0*x	0*y	0*0	0*1	1*x	1*y	1*0	1*1
	<36,16>	<48,28>	<0,0>	<6,4>	<48,28>	<64,49>	<0,0>	<8,7>
	x*x	x*y	x*0	x*1	y*x	y*y	y*0	y*1
	<6,4>	<8,7>	<0,0>	<1,1>	<7,5>	<9,8>	<1,1>	<2,2>
	0+x	0+y	0+0	0+1	1+x	1+y	1+0	1+1
	<12,8>	<14,11>	<6,4>	<7,5>	<14,11>	<16,14>	<8,7>	<9,8>
height 1:	x+x	x+y	x+0	x+1	y+x	y+y	y+0	y+1
	<6,4>	<8,7>	<0,0>	<1,1>				
height 0:	x	y	0	1				

$$S \to E$$
$$E \to E + E$$
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$$E \to [EList]$$
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[Albarghouthi et al. 2013, Udupa et al. 2013]

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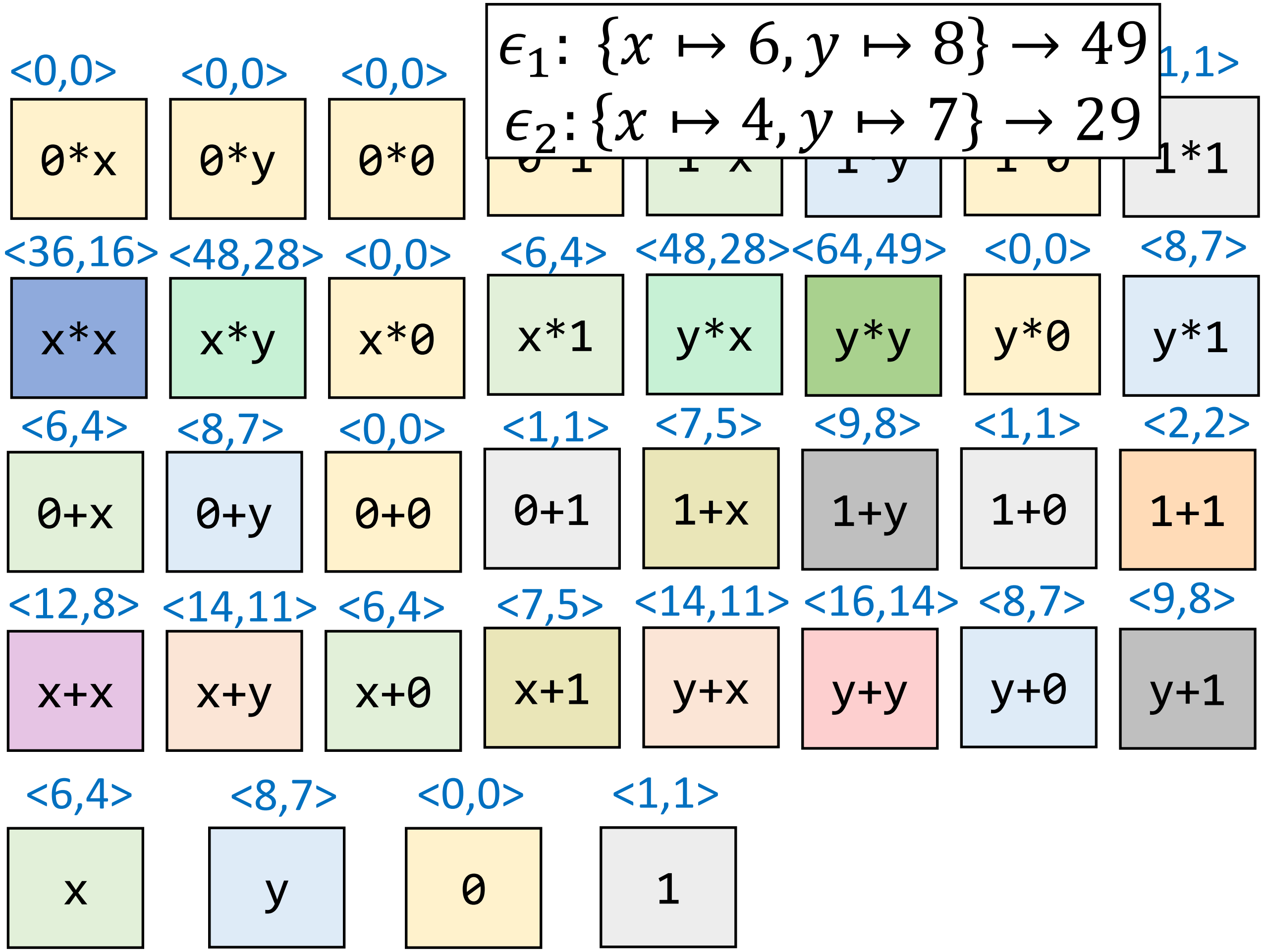
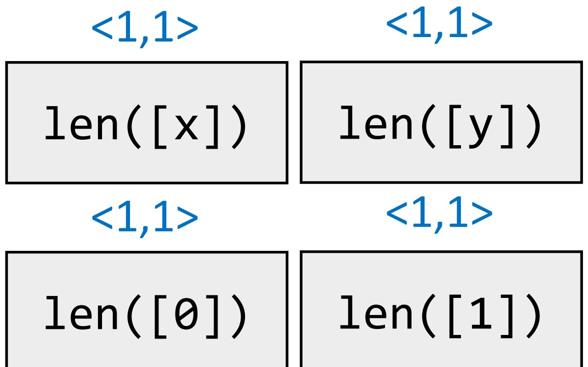


	<0,0>	<0,0>	<0,0>					1,1>
	0*x	0*y	0*0	0*1	1*x	1*y	1*0	1*1
	<36,16>	<48,28>	<0,0>	<6,4>	<48,28>	<64,49>	<0,0>	<8,7>
	x*x	x*y	x*0	x*1	y*x	y*y	y*0	y*1
	<6,4>	<8,7>	<0,0>	<1,1>	<7,5>	<9,8>	<1,1>	<2,2>
	0+x	0+y	0+0	0+1	1+x	1+y	1+0	1+1
	<12,8>	<14,11>	<6,4>	<7,5>	<14,11>	<16,14>	<8,7>	<9,8>
height 1:	x+x	x+y	x+0	x+1	y+x	y+y	y+0	y+1
	<6,4>	<8,7>	<0,0>	<1,1>				
height 0:	x	y	0	1				



<1,1>	<1,1>
len([x])	len([y])
<1,1>	<1,1>
len([0])	len([1])

Observational equivalence

[Albarghouthi et al. 2013, Udupa et al. 2013]

Equivalence:

$p_1 \equiv p_2$ i.f.f. for every possible input i ever, $[\![p_1]\!](i) = [\![p_2]\!](i)$

Observational equivalence:

$p_1 \equiv_{OE} p_2$ i.f.f. for every input i the user cares about, $[\![p_1]\!](i) = [\![p_2]\!](i)$

Equivalence classes

[Albarghouthi et al. 2013, Udupa et al. 2013]

$$\epsilon_1\!: \{x \mapsto 6, y \mapsto 8\} \to 49$$
$$\epsilon_2\!: \{x \mapsto 4, y \mapsto 7\} \to 29$$

	<0,0> 0*x	<0,0> 0*y	<0,0> 0*0	0*1	1*x	1*y	1*0	1,1> 1*1
	<36,16> x*x	<48,28> x*y	<0,0> x*0	<6,4> x*1	<48,28> y*x	<64,49> y*y	<0,0> y*0	<8,7> y*1
	<6,4> 0+x	<8,7> 0+y	<0,0> 0+0	<1,1> 0+1	<7,5> 1+x	<9,8> 1+y	<1,1> 1+0	<2,2> 1+1
height 1:	<12,8> x+x	<14,11> x+y	<6,4> x+0	<7,5> x+1	<14,11> y+x	<16,14> y+y	<8,7> y+0	<9,8> y+1
height 0:	<6,4> x	<8,7> y	<0,0> 0	<1,1> 1				

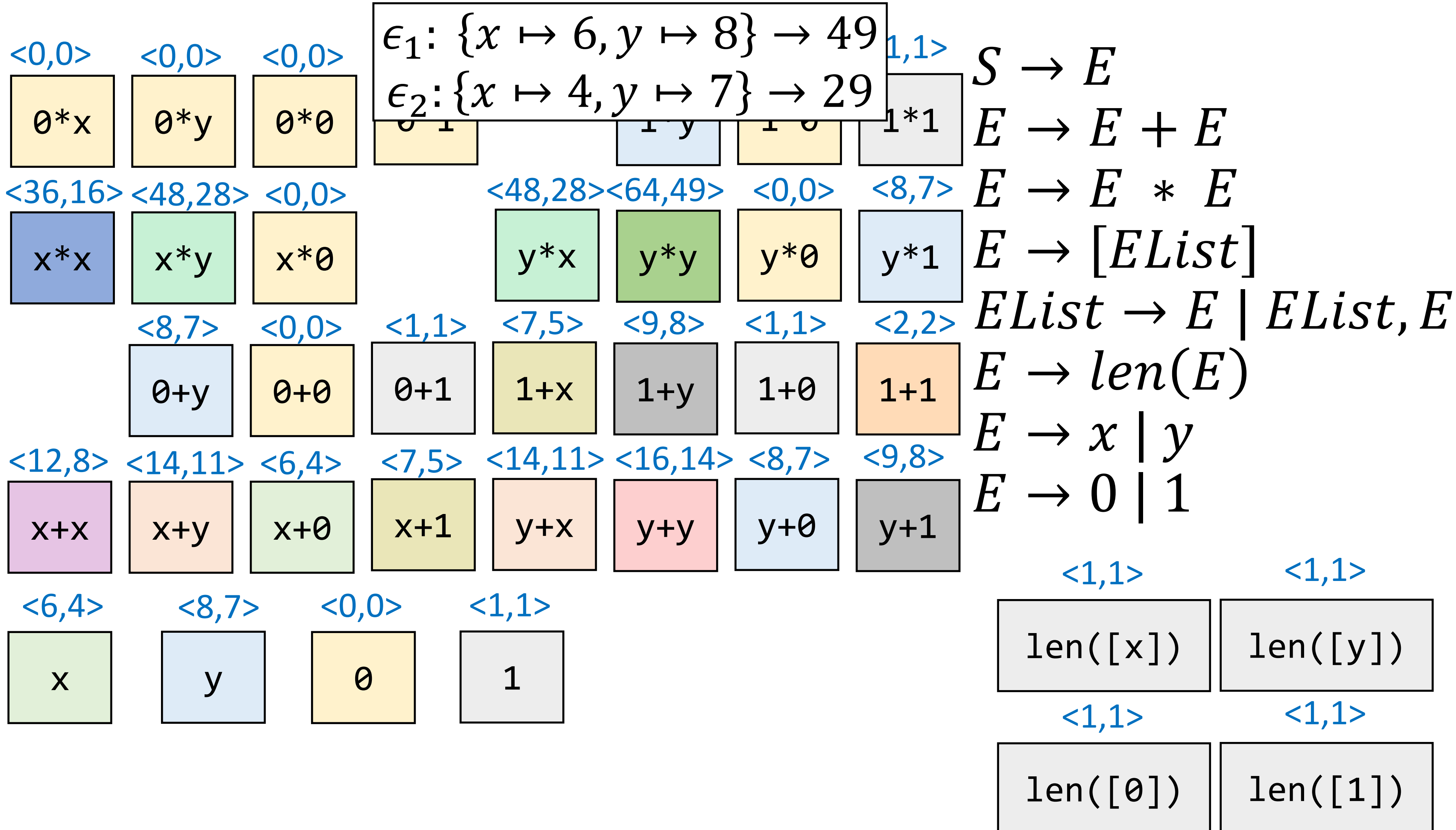
$$
\begin{aligned}
S &\to E \\
E &\to E + E \\
E &\to E * E \\
E &\to [EList] \\
EList &\to E \mid EList, E \\
E &\to len(E) \\
E &\to x \mid y \\
E &\to 0 \mid 1
\end{aligned}
$$

<1,1> len([x])	<1,1> len([y])
<1,1> len([0])	<1,1> len([1])

Equivalence classes

[Albarghouthi et al. 2013, Udupa et al. 2013]

ϵ_1: $\{x \mapsto 6, y \mapsto 8\} \to 49$
ϵ_2: $\{x \mapsto 4, y \mapsto 7\} \to 29$



<0,0> 0*x | <0,0> 0*y | <0,0> 0*0 | 0*1 | 1*y | 1*0 | 1,1> 1*1

<36,16> x*x | <48,28> x*y | <0,0> x*0 | <48,28> y*x | <64,49> y*y | <0,0> y*0 | <8,7> y*1

<8,7> 0+y | <0,0> 0+0 | <1,1> 0+1 | <7,5> 1+x | <9,8> 1+y | <1,1> 1+0 | <2,2> 1+1

height 1: <12,8> x+x | <14,11> x+y | <6,4> x+0 | <7,5> x+1 | <14,11> y+x | <16,14> y+y | <8,7> y+0 | <9,8> y+1

height 0: <6,4> x | <8,7> y | <0,0> 0 | <1,1> 1

<1,1> len([x]) | <1,1> len([y])

<1,1> len([0]) | <1,1> len([1])

$$S \to E$$
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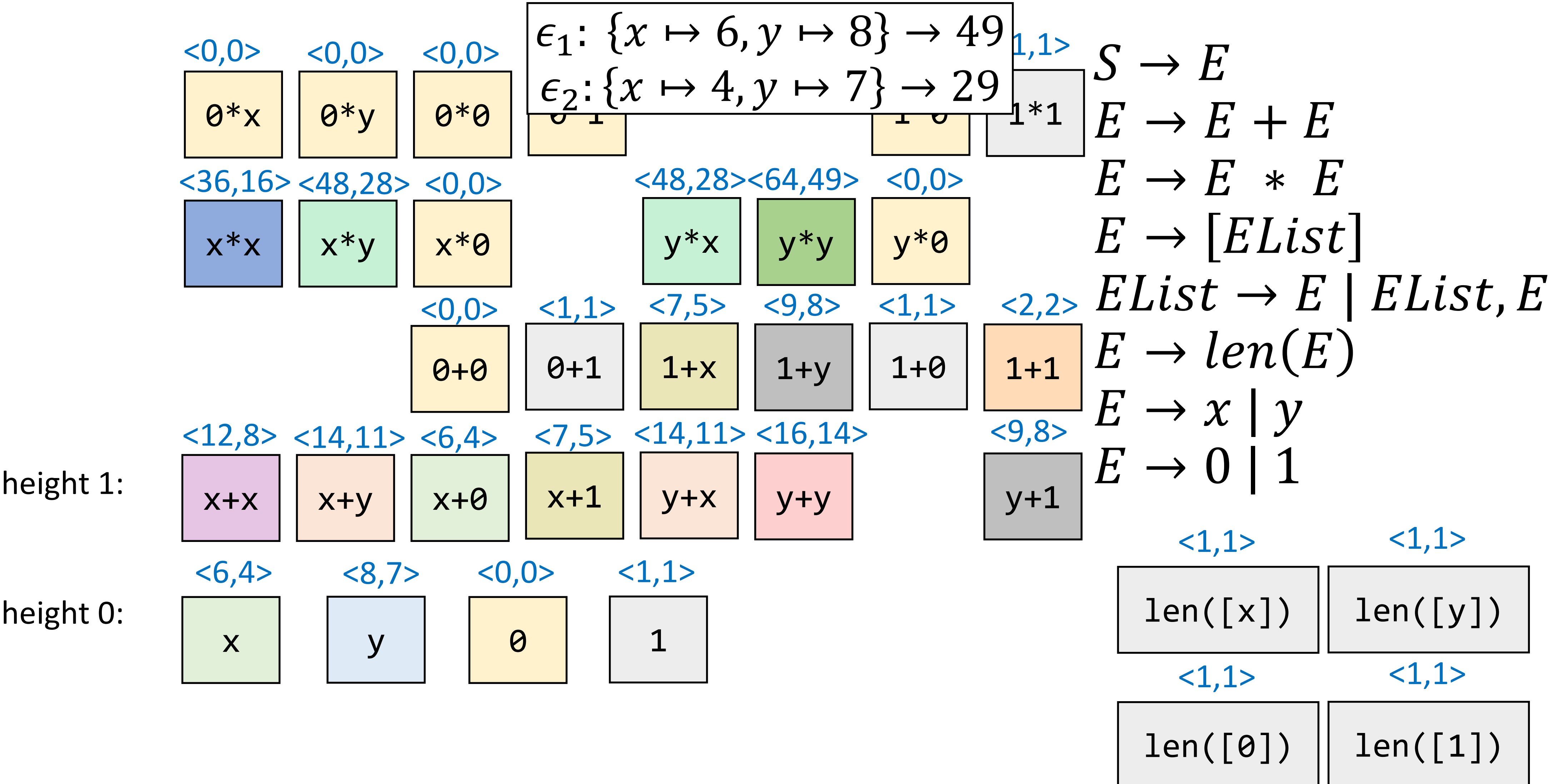

Equivalence classes

[Albarghouthi et al. 2013, Udupa et al. 2013]

$$\epsilon_1\text{: } \{x \mapsto 6, y \mapsto 8\} \to 49$$
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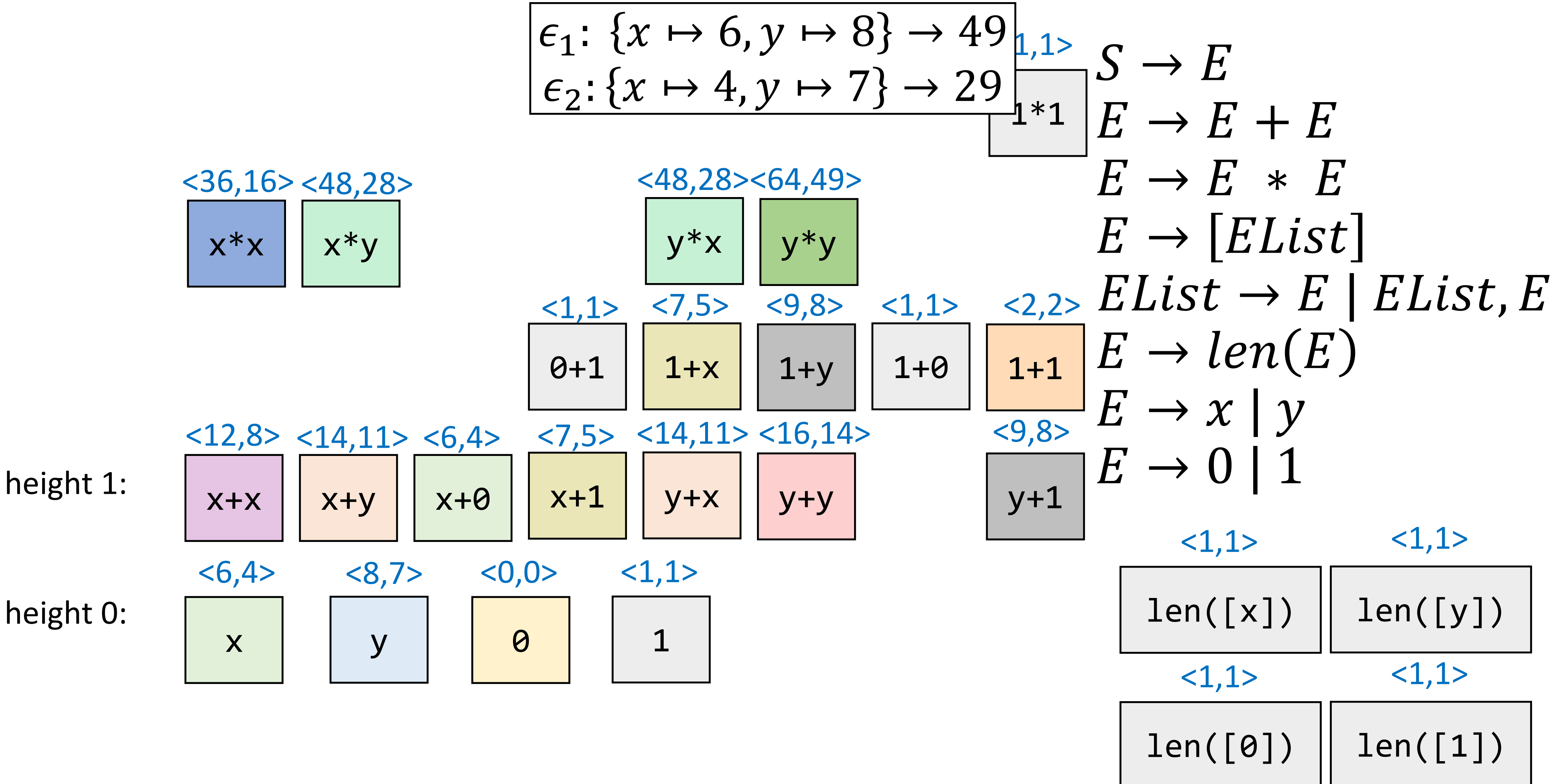

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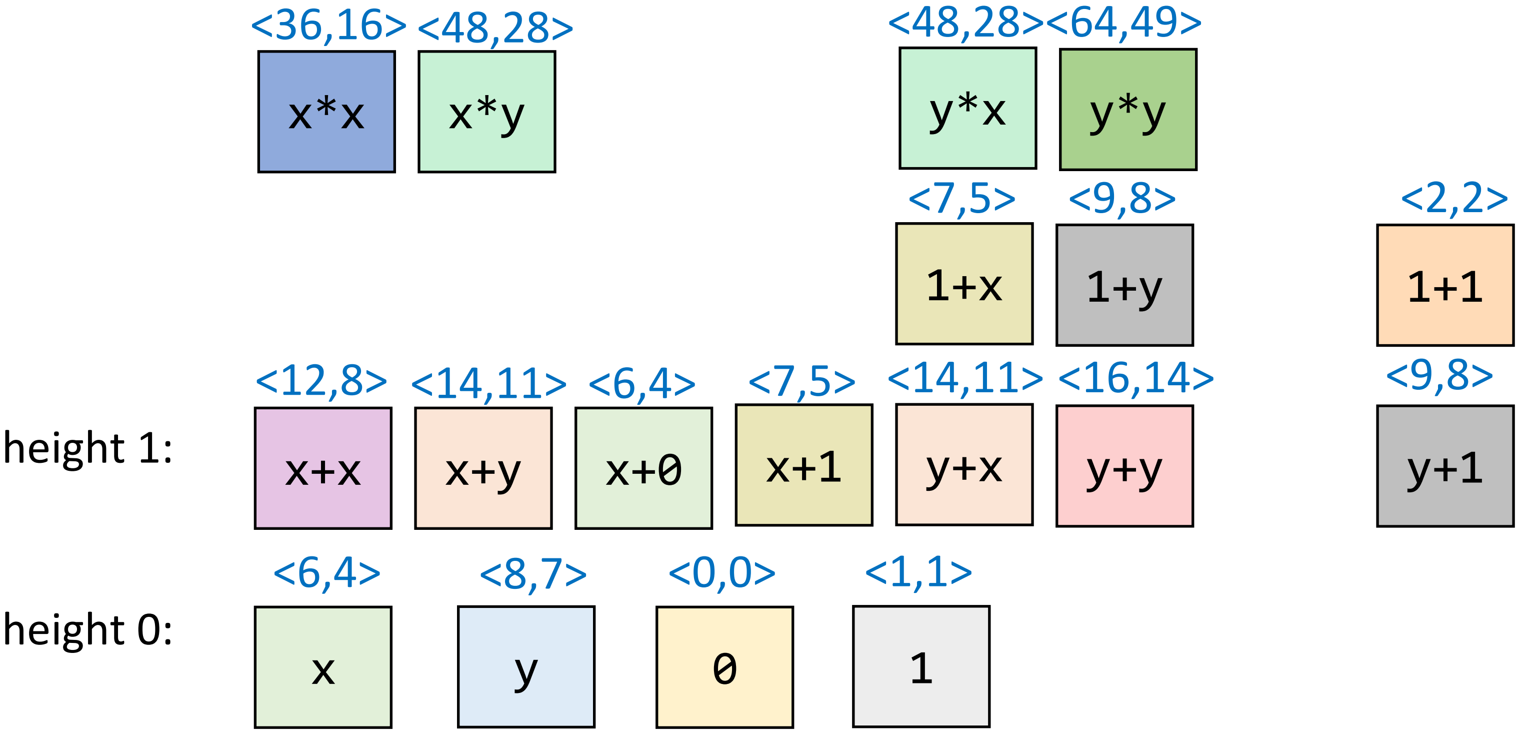
Equivalence classes

[Albarghouthi et al. 2013, Udupa et al. 2013]

$$\boxed{\begin{array}{l} \epsilon_1\text{: } \{x \mapsto 6, y \mapsto 8\} \to 49 \\ \epsilon_2\text{: } \{x \mapsto 4, y \mapsto 7\} \to 29 \end{array}}$$



<36,16> x*x <48,28> x*y

<48,28> y*x <64,49> y*y

<7,5> 1+x <9,8> 1+y

<2,2> 1+1

height 1: <12,8> x+x <14,11> x+y <6,4> x+0 <7,5> x+1 <14,11> y+x <16,14> y+y

<9,8> y+1

height 0: <6,4> x <8,7> y <0,0> 0 <1,1> 1

$$S \to E$$
$$E \to E + E$$
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Equivalence classes

[Albarghouthi et al. 2013, Udupa et al. 2013]

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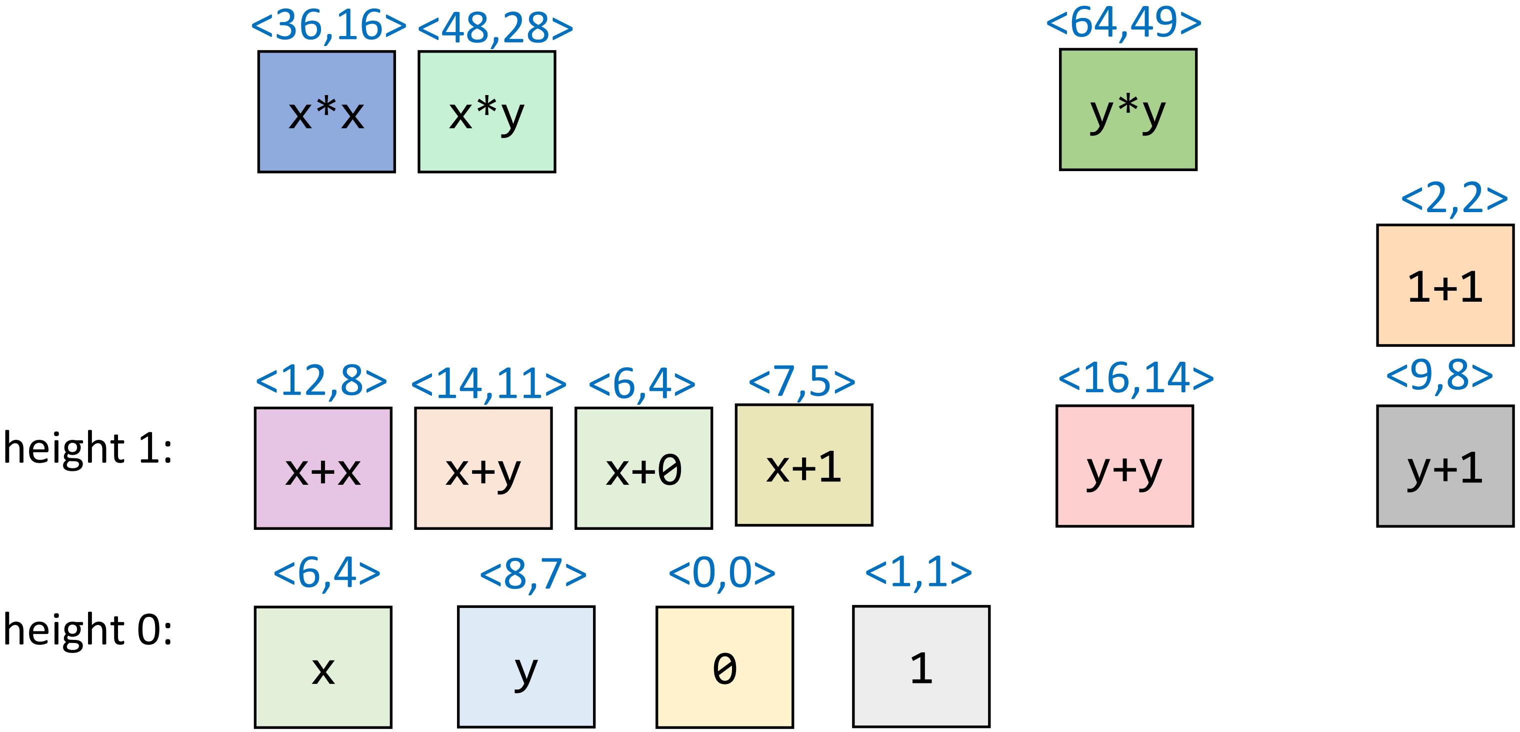

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<16,14> y+y
<9,8> y+1

height 0:
<6,4> x
<8,7> y
<0,0> 0
<1,1> 1

On the fly

[Albarghouthi et al. 2013, Udupa et al. 2013]

$$\epsilon_1\colon \{x \mapsto 6, y \mapsto 8\} \to 49$$
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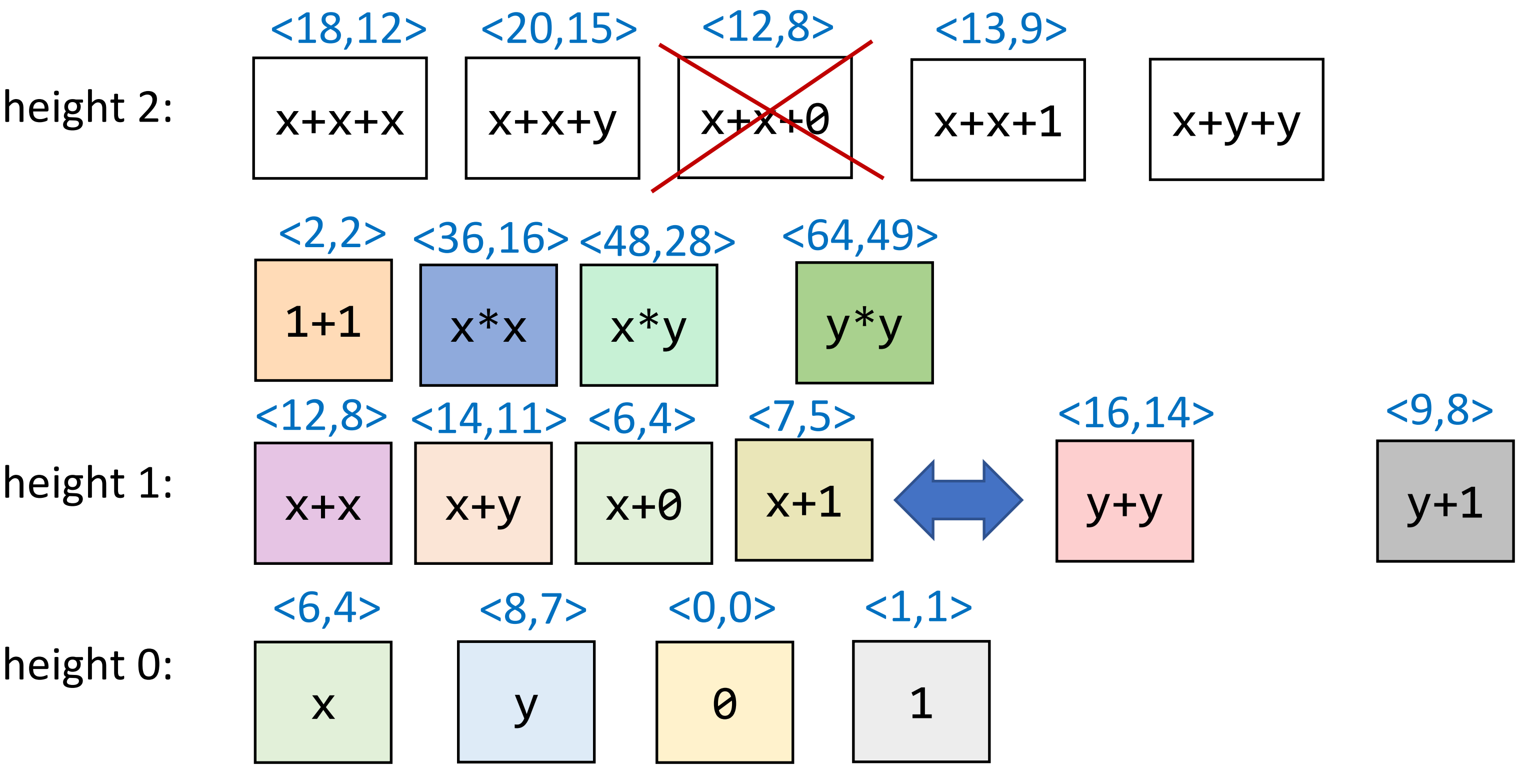

$$S \to E$$
$$E \to E + E$$
$$E \to E \ * \ E$$
$$E \to [EList]$$
$$EList \to E \mid EList, E$$
$$E \to len(E)$$
$$E \to x \mid y$$
$$E \to 0 \mid 1$$

Pros and cons of bottom-up

Pros:

- Iterative deepening is free
- No need to solver-encode anything

Cons:

- Have to specify literals
- Purity
- OE is very aggressive

Summary

- This time:
  - Program synthesis
  - Types of specification
  - Search algorithms
  - Directions for enumerative search
- Next time:
  - What happens when you give this to people?